

3-Phase Induction Motors

* Classification Of A.C. Motors:

Different a.c. motors may, however, be classified and divided into various groups from the following different points of view:

(A) As Regards their Principle of Operation

(1) Synchronous motors

- (i) plain synchronous motors
- (ii) Super Synchronous motors

(2) Asynchronous motors

(a) Induction motors

- (i) Squirrel cage induction motors $\rightarrow$
 - Single cage I.M
 - Double cage I.M
- (ii) Slip-Ring induction motors

(b) Commutator motors

- (i) Series motors $\rightarrow$
 - Single-phase motors
 - Universal motors
- (ii) Compensated motors $\rightarrow$
 - Conductively
 - Inductively
- (iii) Shunt motors $\rightarrow$
 - Simple
 - Compensated
- (iv) Repulsion motors $\rightarrow$
 - straight
 - Compensated
- (v) Repulsion-start induction motor
- (vi) Repulsion induction motor

(B) As Regards the type of current

- (1) Single-phase
- (2) Three-phase

(C) As Regards their Speed

- (1) Constant speed
- (2) Variable speed
- (3) Adjustable speed.

(D) As Regards their structural features

- (1) open
- (2) enclosed
- (3) Semi closed
- (4) ventilated
- (5) pipe-ventilated
- (6) Risered-frame eye. etc....

* Induction Motor:

General principle:

In d.c. motors, the electric power is conducted directly to the armature (i.e., rotating part) through brushes and commutator. Hence in this sense, a d.c. motor can be called as conduction motor. However, in a.c. motors, the rotor does not receive electric power by conduction but by induction in exactly the same way as the secondary of a 2-winding transformer receives its power from the primary. That's why such motors are known as "induction" motors. In fact, an induction motor can be treated as a "rotating transformer", i.e., one in which primary winding is stationary but the secondary is free to rotate.

Of all the a.c. motors, the poly phase induction motor is the one which is extensively used for various kinds of industrial drives. It has the following main advantages and also some disadvantages.

Advantages:

1. It has very simple & extremely rugged, almost unbreakable construction.
2. Its cost is low and it is very reliable.
3. It has sufficiently high efficiency.
4. It requires minimum of maintenance.
5. It starts up from rest and needs no extra starting motor and has not to be synchronised. (i.e., self starting).

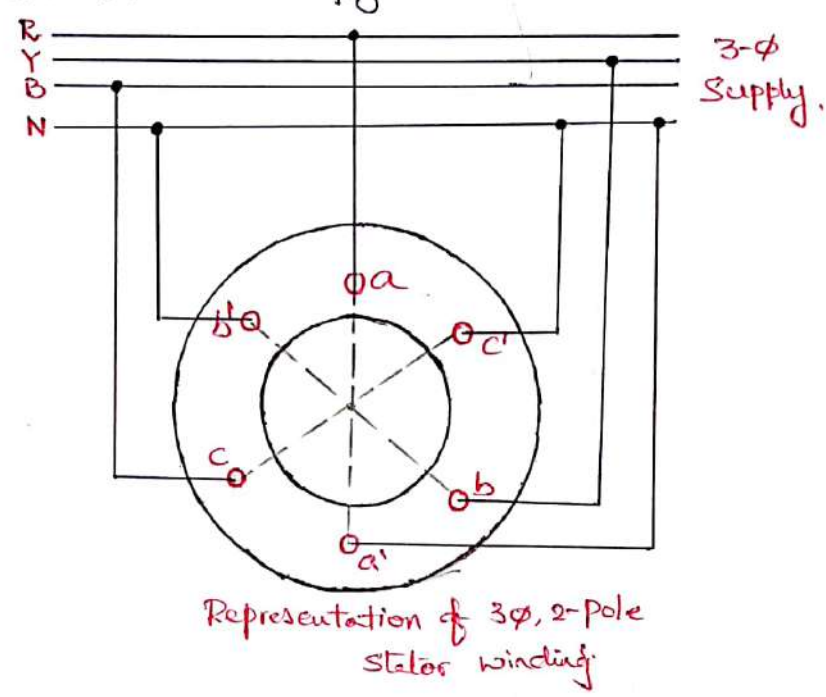
Disadvantages:

1. Its speed cannot be varied without sacrificing some of its efficiency.
2. Just like a d.c. shunt motor, its speed decreases with increase in load.
3. Its starting torque is somewhat inferior to that of a d.c. shunt motor.

* Production of Rotating Magnetic field:

When the three-phase uniformly distributed winding suitably wound on the stator supplied with 3- ϕ a.c. supply, a rotating magnetic field of constant magnitude rotate at synchronous speed is produced.

Consider a 3- ϕ , 2-pole stator having three similar windings aa' , bb' & cc' displaced in space by 120° , supplied with 3- ϕ ac supply as shown in below figure.

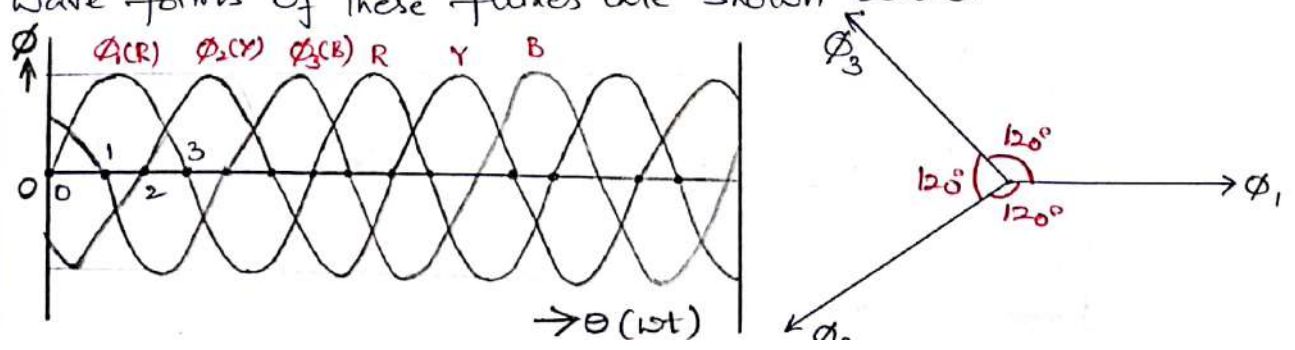


Due to current flowing in each phase of the stator produces an alternating flux which is assumed to be sinusoidal in nature. Let these fluxes be ϕ_1 , ϕ_2 and ϕ_3 and are displaced from each other by 120° .

The instantaneous values of fluxes w.r.t. time is given by

$$\begin{aligned}\phi_1 &= \phi_m \sin \theta \\ \phi_2 &= \phi_m \sin (\theta - 120^\circ) \text{ \& } \\ \phi_3 &= \phi_m \sin (\theta - 240^\circ)\end{aligned}$$

Where ϕ_m = maximum value of the flux due to any phase. The wave forms of these fluxes are shown below.



This three fluxes together gives rise to an effect of rotating flux called as "Rotating Magnetic field."

(i) When $\theta = 0^\circ$ (at point 'O'):

$$\phi_1 = \phi_m \sin 0^\circ = 0$$

$$\phi_2 = \phi_m \sin(0 - 120^\circ) = -\frac{\sqrt{3}}{2} \phi_m$$

$$\phi_3 = \phi_m \sin(0 - 240^\circ) = \frac{\sqrt{3}}{2} \phi_m$$

The resultant flux of $\phi_2 \times \phi_3$ is

$$\phi_r = \sqrt{\left(-\frac{\sqrt{3}}{2} \phi_m\right)^2 + \left(\frac{\sqrt{3}}{2} \phi_m\right)^2 - 2\left(-\frac{\sqrt{3}}{2} \phi_m\right)\left(\frac{\sqrt{3}}{2} \phi_m\right) \cos(-\phi_2, \phi_3)}$$

$$\phi_r = \sqrt{\frac{3}{4} \phi_m^2 + \frac{3}{4} \phi_m^2 + \frac{3}{2} \phi_m^2 \cdot \cos 60^\circ}$$

$$\phi_r = \sqrt{\frac{3}{2} \phi_m^2 + \frac{3}{2} \phi_m^2 \cdot \frac{1}{2}} = 1.5 \phi_m$$

$$\boxed{\phi_r = 1.5 \phi_m}$$

(ii) When $\theta = 60^\circ$ (at point '1'):

$$\phi_1 = \phi_m \sin 60^\circ = \frac{\sqrt{3}}{2} \phi_m$$

$$\phi_2 = \phi_m \sin(60^\circ - 120^\circ) = -\frac{\sqrt{3}}{2} \phi_m$$

$$\phi_3 = \phi_m \sin(60^\circ - 240^\circ) = 0$$

The resultant flux of $\phi_1 \times \phi_2$ is

$$\phi_r = \sqrt{\left(\frac{\sqrt{3}}{2} \phi_m\right)^2 + \left(-\frac{\sqrt{3}}{2} \phi_m\right)^2 - 2\left(\frac{\sqrt{3}}{2} \phi_m\right)\left(-\frac{\sqrt{3}}{2} \phi_m\right) \cos(\phi_1, -\phi_2)}$$

$$\phi_r = \sqrt{\frac{3}{4} \phi_m^2 + \frac{3}{4} \phi_m^2 + 2 \cdot \frac{3}{4} \phi_m^2 \cdot \cos 60^\circ} = \sqrt{\frac{3}{4} \phi_m^2 + \frac{3}{4} \phi_m^2 + \frac{3}{4} \phi_m^2}$$

$$\boxed{\phi_r = 1.5 \phi_m}$$

(iii) When $\theta = 120^\circ$ (at point '2'):

$$\phi_1 = \phi_m \sin 120^\circ = \frac{\sqrt{3}}{2} \phi_m$$

$$\phi_2 = \phi_m \sin(120^\circ - 120^\circ) = 0$$

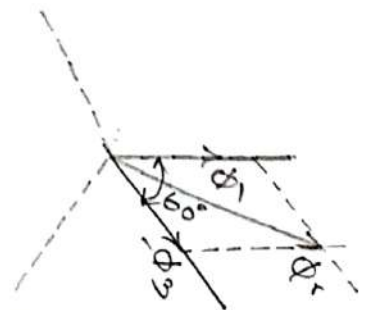
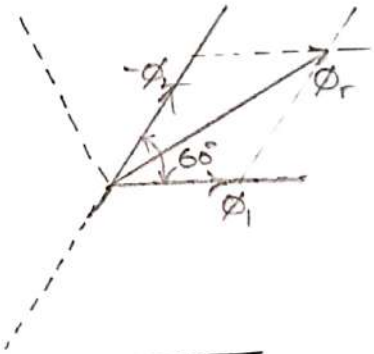
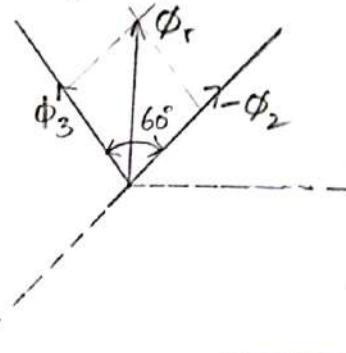
$$\phi_3 = \phi_m \sin(120^\circ - 240^\circ) = -\frac{\sqrt{3}}{2} \phi_m$$

The resultant flux of $\phi_1 \times \phi_3$ is

$$\phi_r = \sqrt{\left(\frac{\sqrt{3}}{2} \phi_m\right)^2 + \left(-\frac{\sqrt{3}}{2} \phi_m\right)^2 - 2\left(\frac{\sqrt{3}}{2} \phi_m\right)\left(-\frac{\sqrt{3}}{2} \phi_m\right) \cos(\phi_1, -\phi_3)}$$

$$\phi_r = \sqrt{\frac{3}{4} \phi_m^2 + \frac{3}{4} \phi_m^2 + 2 \cdot \frac{3}{4} \phi_m^2 \cos 60^\circ} = \sqrt{\frac{3}{4} \phi_m^2 + \frac{3}{4} \phi_m^2 + \frac{3}{4} \phi_m^2}$$

$$\boxed{\phi_r = 1.5 \phi_m}$$



(iv) When $\theta = 180^\circ$ (at point 3):

$$\phi_1 = \phi_m \sin 180^\circ = 0$$

$$\phi_2 = \phi_m \sin (120^\circ + 180^\circ) = \frac{\sqrt{3}}{2} \phi_m$$

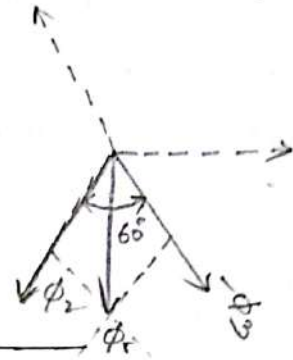
$$\phi_3 = \phi_m \sin (180^\circ - 240^\circ) = -\frac{\sqrt{3}}{2} \phi_m$$

The resultant flux of ϕ_2 & ϕ_3 is

$$\phi_r = \sqrt{\left(\frac{\sqrt{3}}{2} \phi_m\right)^2 + \left(-\frac{\sqrt{3}}{2} \phi_m\right)^2 - 2\left(\frac{\sqrt{3}}{2} \phi_m\right)\left(-\frac{\sqrt{3}}{2} \phi_m\right) \cos (\phi_2, -\phi_3)}$$

$$\phi_r = \sqrt{\frac{3}{4} \phi_m^2 + \frac{3}{4} \phi_m^2 + 2 \cdot \frac{3}{4} \phi_m^2 \cos 60^\circ} = \sqrt{\frac{3}{4} \phi_m^2 + \frac{3}{4} \phi_m^2 + \frac{3}{4} \phi_m^2}$$

$$\phi_r = 1.5 \phi_m$$



From the above, it is clear that

(i) The resultant flux is of constant magnitude and having value $1.5 \phi_m$, i.e., $\phi_r = 1.5 \phi_m$

(ii) The resultant flux rotates at synchronous speed, N_s given by

$$N_s = \frac{120f}{P} \text{ rpm.}$$

$f \rightarrow$ frequency

$P \rightarrow$ No. of poles of stator winding.

"When 3- ϕ supply is fed to 3- ϕ stator winding, a rotating magnetic field of constant magnitude rotating at synchronous speed produced."

* Construction of Induction Motor:

An induction motor consists of two main parts:

(i) Stator (Stationary part)

(ii) Rotor (Rotating part).

(i) Stator:

The stator construction is similar to the stator of a synchronous machine. The stator core is laminated and is slotted on its inner cylindrical surface. These slots house the 3- ϕ stator winding.

The large size motors use open type slots, so that form-wound and properly insulated coils can be placed in them. But small size motors are provided with semiclosed type slots to make the air gap reluctance minimum and to prevent excessive tooth loss.

The 3- ϕ stator winding connected either in star or Delta. It is wound for a definite no. of poles depends upon the speed requirement. Greater the no. of poles, lesser the speed and vice versa. This winding when excited by a 3- ϕ supply produces a rotating magnetic field. The ventilating ducts are provided for the cooling purpose.

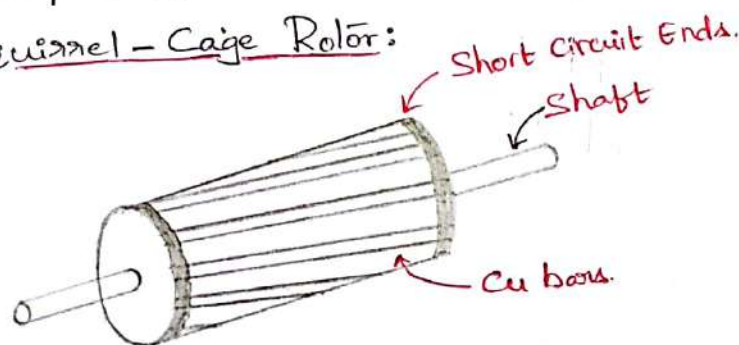
In some cases, all the six terminals of 3- ϕ winding are brought out so that user can connect them either in star or Delta.

(2) Rotor:

The two types of rotor construction are generally used for the induction motor, namely:

- (i) Squirrel-Cage rotor.
- (ii) Slip-Ring rotor.

(i) Squirrel-Cage Rotor:



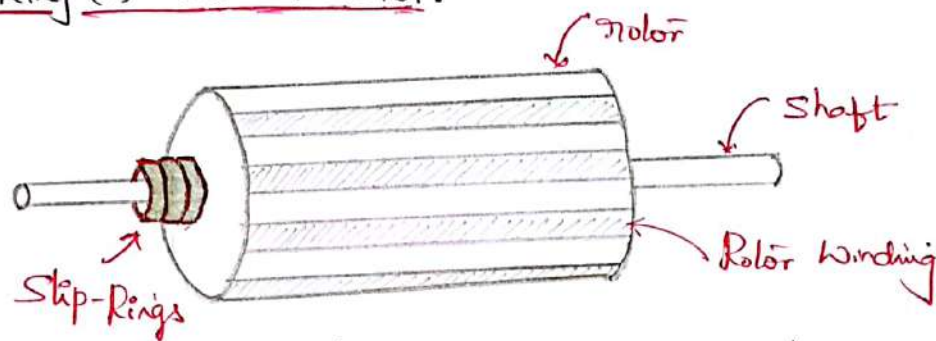
In this type, of rotor consists of an insulated cu or Aluminium bars placed in the slots. These bars are permanently short circuited at both ends by conducting end rings. It is called as squirrel cage rotor because the entire structure looks like a cage. The cage winding is adaptable to any number of poles.

The Squirrel Cage rotor is Cheap & Robust. However its starting torque is low because the resistance of the rotor is very small & there is no way to insert external resistance as the winding is permanently short circuited.

Skewed rotor slots: The rotor slots are not always made parallel to the rotor shaft axis but are slightly skewed with certain angle. This arrangement gives following advantages.

- (i) It reduces the magnetic noise during operation of the motor.
- (ii) It helps in producing uniform torque.
- (iii) It reduces Cogging, which is nothing but magnetic locking of the rotor with the stator.

(2) Slip-Ring (or) Wound Rotor:



A wound rotor has a laminated core with slots on its outer surface. These slots carry 3-phase rotor winding, which is similar to the stator winding. Both the stator and rotor windings are designed for the same number of poles.

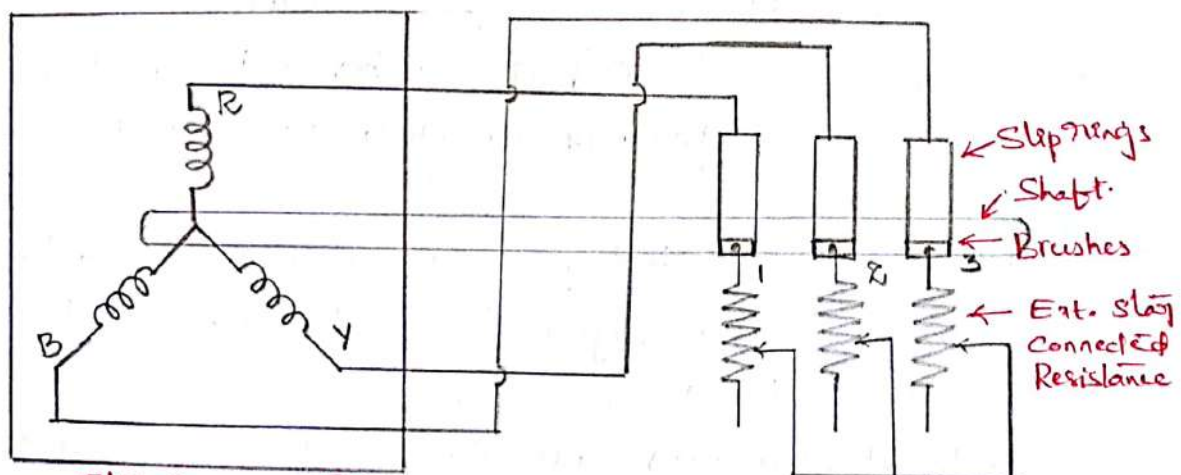
The 3- ϕ rotor winding is usually star connected. The three ends of the rotor winding are connected to the slip rings mounted on the rotor shaft. So in this type of motor, the external resistance can be added with the help of brushes and slip ring arrangement in series with each phase of the rotor winding. This arrangement is shown below.

At the time of starting, additional resistance in the rotor circuit is included to have higher starting torque. When the motor is in running condition, the slip rings are short circuited (i.e., the external resistances are completely cut out from the

rotor circuit) by means of a metal collar which is pushed along the shaft and connects all the slip-rings together. At the same time, brushes also lifted from the slip rings to reduce wear and tear of the brushes due to friction.

Comparison of Squirrel Cage & Slip-ring Induction Motors:

S.No.	Squirrel Cage I.M.	Slip-ring I.M.
1.	Rotor Consists of bars which are shorted at both ends by end rings.	Rotor Consists of 3- ϕ winding similar to the stator winding.
2.	Construction is very simple.	Construction is complicated.
3.	External resistance cannot be added in the rotor circuit hence starting torque is less.	External resistance can be added in the rotor circuit hence high starting torque can be achieved.
4.	Most Commonly used	Used for special purpose.
5.	Its cost is less	It is costly.
6.	Mechanically robust, rugged and maintenance free	Requires maintenance
7.	Speed control by rotor resistance is not possible.	Speed control by rotor - resistance is possible.
8.	Rotor Cu losses are less hence efficiency is high.	Efficiency is less.

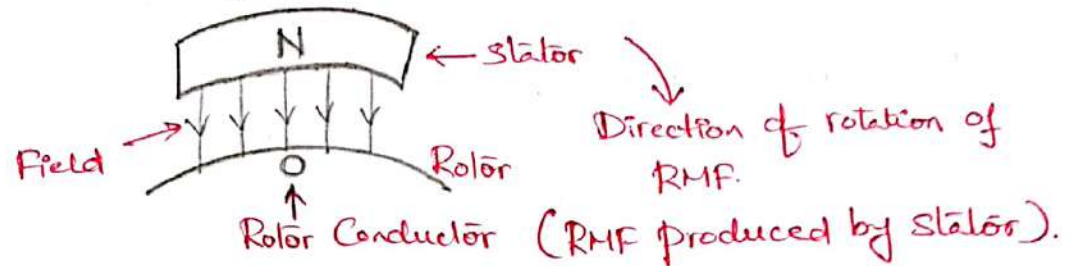


Squirrel-Cage Rotor

Slip-Ring Rotor

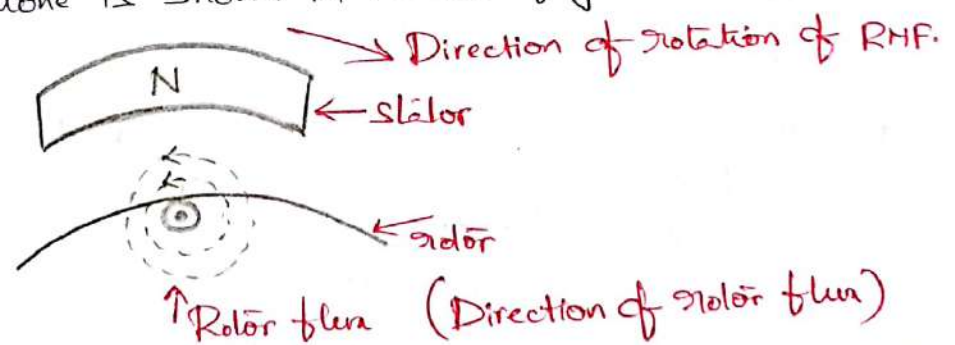
* Principle of Working:

Basically induction motor works on the principle of Induction. When 3- ϕ Supply is given to the 3- ϕ Stator winding, a rotating field of constant magnitude and rotating at synchronous speed is produced. Let direction of rotation of this field is clockwise as shown in below figure.

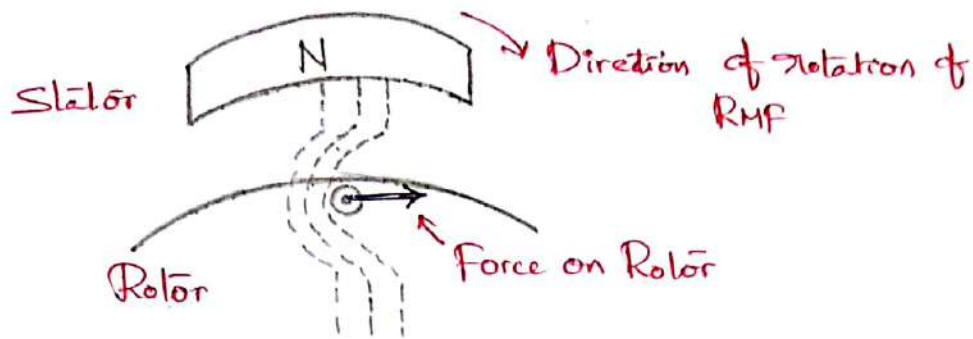


This rotating flux cuts the stationary rotor conductors. So there exists a relative motion or speed between the rotating magnetic field and the rotor conductors, an emf is induced in the rotor according to the "Faraday's laws of electromagnetic induction".

The induced emf in the rotor drives a current through rotor conductors and its direction can be determined by the "Fleming's Right hand rule". For the case considered, direction of rotor current in a conductor is towards the observer. The current carrying rotor conductors produce their own flux called as rotor flux. Hence by applying right hand thumb rule, the direction of rotor flux alone is shown in below figure.



The interaction of stator and rotor fluxes, the rotor conductors (according to the left hand rule) experiences a force, the overall rotor experiences a torque (shown in below figure). Hence rotor starts rotating in the same direction as that of the stator rotating field and tries to catch up the speed of rotating magnetic field.



(Interaction of Two fluxes.)

As the rotor starts rotating, the magnitude of motor induced emf (proportional to relative speed b/w stator rotating field and rotor speed) is reduced.

Let us assume that the rotor rotates at synchronous speed. So the emf induced in the rotor conductors would become zero. and consequently there is no torque on the rotor causing the rotor to fall back in speed. Once the rotor falls back in speed, the relative speed increases and again rotor picks up the speed. But due to inertia of rotor, this does not happen in practice and rotor continues to rotate with a speed slightly less than the synchronous speed of the rotating field.

The induction motor never rotates at synchronous speed. The speed at which it rotates is hence called as sub synchronous speed and motor called as "Asynchronous motor".

* Slip:

The slip is defined as the difference between the synchronous speed (N_s) and the actual speed of the motor (N), expressed as a fraction of synchronous speed.

$$\text{Slip speed} = N_s - N$$

$$\text{Thus slip, } S = \frac{N_s - N}{N_s}$$

$$\text{Percentage slip, } \%S = \frac{N_s - N}{N_s} \times 100$$

The actual speed of the motor can be expressed, from the above equation

$$SN_s = N_s - N$$

$$\Rightarrow N = N_s - SN_s = N_s(1-s)$$

$$N = N_s(1-s)$$

From the slip speed, it may be noticed that the relative speed (or) slip speed ($N_s - N$) of motor and rotating magnetic field is directly proportional to the slip.

$$S \propto (N_s - N)$$

At start (or) stand still, $N=0$.

$$\therefore \text{slip}, S = \frac{N_s - 0}{N_s} = 1 \Rightarrow S=1$$

Slip can not be zero because when $S=0$, $N=N_s$ and the torque is zero.

At no-load slip is about 1% and at full load it is about 3 to 5%. The variation in slip and therefore speed, from no-load to full-load is very small. Thus a 3- ϕ induction motor is known as a "Constant Speed Motor."

Prob: A 6-pole, 50Hz induction motor has a slip of 2.5%. Find its actual speed and slip speed.

Sol: Given that poles, $P=6$

frequency, $f=50\text{Hz}$

$$\text{slip}, S = 2.5\% = 0.025$$

$$\text{Synchronous Speed}, N_s = \frac{120f}{P} = \frac{120 \times 50}{6} = 1000 \text{ rpm.}$$

$$\text{Actual Speed of Motor}, N = N_s(1-s) = 1000(1-0.025) = 975 \text{ rpm.}$$

$$\text{Slip Speed} = N_s - N = 1000 - 975 = 25 \text{ rpm.}$$

Prob: An 8-pole, 50Hz induction motor has a full speed of 725 rpm. What is the percentage slip at full load.

Sol: Given that $P=8$

$f=50\text{Hz}$ and full speed, $N=725 \text{ rpm.}$

$$\text{Synchronous Speed}, N_s = \frac{120f}{P} = \frac{120 \times 50}{8} = 750 \text{ rpm.}$$

$$\% \text{ Slip} = \frac{N_s - N}{N_s} \times 100 = \frac{750 - 725}{750} \times 100 = \frac{25}{750} \times 100 = 3.33\%$$

Prob.

Two 440V, 50Hz, 4-pole, 3- ϕ Induction motors, running at 950 rpm and 715 rpm respectively. Determine which of the two motors is running at higher slip.

* Effect of slip on Rotor Parameters:

(1) Rotor Frequency:

The relation between Synchronous Speed, Supply frequency and the number of poles is given by

$$f = \frac{N_s P}{120} \text{ Hz} \quad \text{--- (1)}$$

When the rotor rotates at a speed 'N', then the rotor conductors cut the rotating magnetic field at the relative speed, i.e., $(N_s - N)$. The frequency of the rotor induced emf or current ' f_r ' can be expressed as,

$$f_r = \frac{(N_s - N) \times P}{120} \text{ Hz} \quad \text{--- (2)}$$

From (1) & (2)

$$\frac{(2)}{(1)} = \frac{f_r}{f} = \frac{(N_s - N) \times P / 120}{N_s \times P / 120} = \frac{N_s - N}{N_s} = S.$$

$$\boxed{f_r = S \times f} \rightarrow \text{in Running Condition.}$$

So rotor frequency in running condition is slip times the supply frequency.

At start/standstill, $N=0$, $\text{Slip}=1$

$$\text{Hence } \boxed{f_r = f} \rightarrow \text{in Standstill Condition.}$$

(2) Rotor induced EMF:

When the rotor is stationary or standstill, i.e., $\text{Slip}=1$, the emf induced in the rotor is maximum. This is due to the rate of cutting flux by rotor is maximum as the relative speed between rotor and rotating magnetic field is maximum.

As rotor gains speed, the relative speed decreases. This reduces the magnitude of induced emf in the rotor, proportionally. The relative speed is " $N_s - N$ " rpm, when the motor is running at speed 'N' rpm.

At stand still:

Rotor induced emf/phase at standstill

$$E_2 \propto N_s \quad \text{--- (3)}$$

Under Running Condition:

Rotor induced emf/phase under running condition

$$E_{2r} \propto (N_s - N) \quad \text{--- (4)}$$

from (3) & (4)

$$\frac{(4)}{(3)} \Rightarrow \frac{E_{2r}}{E_2} = \frac{N_s - N}{N_s} = s$$

$$\boxed{E_{2r} = s E_2}$$

"Rotor induced emf under running condition will be slip times the Rotor induced emf at standstill."

(3) Rotor Reactance, Resistance & Impedance:

Rotor Reactance:

The rotor reactance depends upon the frequency. At slip s , the frequency of the rotor induced emf $f_r = sf$

Let L_2 = Inductance of the rotor per phase

At standstill $f_r = f$

$$\text{Rotor Reactance, } X_2 = 2\pi f L_2$$

Under Running condition $f_r = sf$

$$\begin{aligned} \text{Rotor Reactance, } X_{2r} &= 2\pi f_r L_2 = 2\pi s f L_2 \\ &= s [2\pi f L_2] = s X_2 \end{aligned}$$

$$\therefore \boxed{X_{2r} = s X_2}$$

"Rotor Reactance under running condition will be slip times the Rotor Reactance at standstill."

Rotor Resistance:

Let R_2 = standstill rotor Resistance/phase

Rotor Resistance is independent of frequency and hence rotor resistance remains same as R_2 Ω /ph at standstill and in running conditions.

Rotor Impedance:

The Rotor Impedance/phase at standstill and under running condition is given by

$$Z_2 = R_2 + jX_2 = \sqrt{R_2^2 + X_2^2} \text{ at standstill}$$

$$Z_2 = R_2 + jSX_2 = \sqrt{R_2^2 + (SX_2)^2} \text{ at slip 's'}$$

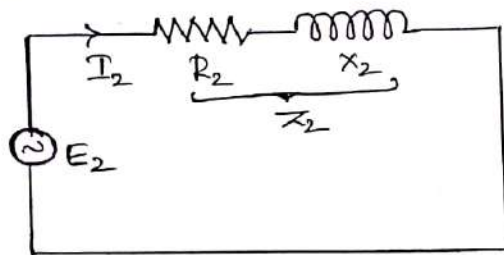
(4) Rotor Current and Power factor:

At standstill: (i.e., $S=1$)

Let I_2 = standstill rotor current/phase

$\cos\phi_2$ = P.f. of the rotor at standstill

Then the equivalent circuit of the rotor at standstill is



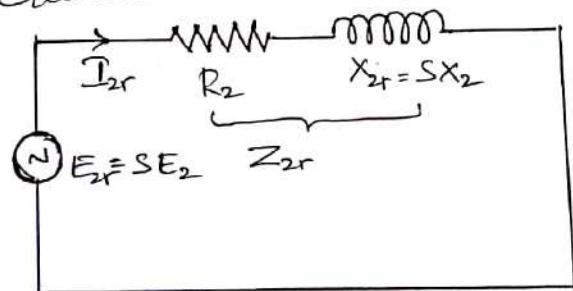
$$\therefore I_2 = \frac{E_2}{Z_2} = \frac{E_2}{\sqrt{R_2^2 + X_2^2}} \text{ and}$$

$$\cos\phi_2 = \frac{R_2}{Z_2} = \frac{R_2}{\sqrt{R_2^2 + X_2^2}}$$

Where E_2 = standstill rotor induced emf/phase.

Under Running Condition (i.e., at slip S):

The equivalent circuit of the rotor under running condition



Rotor Current/phase under running condition

$$I_{2r} = \frac{E_{2r}}{Z_{2r}} = \frac{SE_2}{\sqrt{R_2^2 + (SX_2)^2}} \text{ and}$$

$$\cos\phi_{2r} = \frac{R_{2r}}{Z_{2r}} = \frac{R_2}{\sqrt{R_2^2 + (SX_2)^2}}$$

Prob: A 3- ϕ Induction motor having a star connected rotor has an induced emf of 80V between the slip rings at standstill on open-circuit. The motor has a resistance and reactance per phase of 4Ω & 14Ω respectively. Calculate current per phase and power factor when

(a) slip rings are short circuited

(b) slip-rings are connected to a star connected rheostat of 3Ω /phase.

Sol: (a) When slip rings are short circuited:

Standstill rotor emf = 80V

Standstill rotor induced emf/phase, $E_2 = \frac{80}{\sqrt{3}} = 46.19V$.

Rotor Resistance, $R_2 = 4\Omega/\phi$

Standstill rotor Reactance, $X_2 = 14\Omega/\phi$.

Rotor Impedance at Standstill, $Z_2 = \sqrt{R_2^2 + X_2^2} = \sqrt{4^2 + 14^2} = 14.56\Omega/\phi$.

Rotor Current/phase, $I_2 = \frac{E_2}{Z_2} = \frac{46.19}{14.56} = 3.172A$

Power factor, $\cos\phi_2 = \frac{R_2}{Z_2} = \frac{4}{14.56} = 0.275$ lag.

(b) When Slip Rings are connected to a star connected rheostat of 3Ω /phase:

Rotor Resistance, $R_2 = 4 + 3 = 7\Omega/\text{phase}$

The rotor impedance at this condition,

$$Z_2 = \sqrt{R_2^2 + X_2^2} = \sqrt{7^2 + 14^2} = 15.652\Omega/\text{phase}$$

Rotor Current/phase, $I_2 = \frac{E_2}{Z_2} = \frac{46.19}{15.652} = 2.951A$.

Power factor, $\cos\phi_2 = \frac{R_2}{Z_2} = \frac{7}{15.652} = 0.447$ lagging.

Prob: The star connected rotor of an induction motor has a standstill impedance of $(0.4 + j4)\Omega/\text{ph}$ and rheostat impedance/ph of $(6 + j2)\Omega$. The motor has an induced emf of 80V between slip rings at standstill, when connected to its normal voltage. Find

(i) rotor current at standstill with the rheostat in circuit

(ii) When slip rings are short circuited and motor running at 3% slip.

Sol: (i) At standstill with the rheostat in circuit:

Rotor Resistance, $R_2 = 6\Omega$,

Rotor Inductance, $X_2 = 2\Omega$

Rotor Impedance, $Z_2 = \sqrt{6^2 + 2^2} = 6.325\Omega/\text{ph}$

Standstill rotor induced emf/phase = $E_2 = \frac{80}{\sqrt{3}} = 46.19 \text{ V}$.

Total Resistance, $R_2 = 0.4 + 6 = 6.4 \Omega/\text{phase}$

Total Rotor Reactance, $X_2 = 4 + 2 = 6 \Omega/\text{phase}$

Total Rotor Impedance, $Z_2 = \sqrt{R_2^2 + X_2^2} = \sqrt{6.4^2 + 6^2} = 8.773 \Omega/\text{ph.}$

Rotor current, $I_2 = \frac{E_2}{Z_2} = \frac{46.19}{8.773} = 5.27 \text{ A}$

Power factor, $\cos \phi_2 = \frac{R_2}{Z_2} = \frac{6.4}{8.773} = 0.729 \text{ lagging}$

(ii) When slip rings are short circuited & motor running at 3% slip

Slip, $s = 3\% = 0.03$

Rotor induced emf/phase = $E_{2r} = s E_2 = 0.03 \times 46.19 = 1.386 \text{ V}$

Rotor Impedance/phase = $Z_{2r} = \sqrt{(6.4)^2 + 4^2} = 0.4176 \Omega/\phi$

Rotor current, $I_{2r} = \frac{E_{2r}}{Z_{2r}} = \frac{1.386}{0.4176} = 3.32 \text{ A}$

Power factor, $\cos \phi_{2r} = \frac{R_{2r}}{Z_{2r}} = \frac{0.4}{0.4176} = 0.957 \text{ lagging}$

Prob: A 3- ϕ Slip ring Induction motor gives a reading of 55V across slip rings on open circuit when at rest with normal stator voltage applied. The motor is star connected and has an impedance of $(0.7 + j5) \Omega/\phi$. Find the motor current when the machine is

(a) at standstill with slip rings joined to a star-connected starter with a phase impedance of $(4 + j3) \Omega$ &

(b) running normally with a 5% of slip.

* Polyphase Induction Motors:

A poly phase induction motor is a singly-excited A.C. machine in the sense that it is supplied power from a single A.C. source. Its stator winding is directly connected to A.C. source, whereas its rotor winding receives its energy from stator by means of induction (i.e., transformer action).

The balanced polyphase currents in polyphase windings produce a constant amplitude rotating mmf wave. The stator produced mmf wave and rotor produced mmf wave, both rotate in the air gap in the same direction at synchronous speed. These two m.m.f waves are thus stationary with respect to each other. Consequently the development of steady electromagnetic torque is possible at all speeds but not at synchronous speed. The stator and rotor m.m.f waves combine to give the resultant air-gap flux density wave of constant amplitude and rotating at synchronous speed. Since an induction motor can't run at synchronous speed, it is called "asynchronous machine".

Induction Motor as a Transformer:

An induction motor is similar to a transformer in many aspects. In this connection, consider an induction motor with both its stator and rotor windings in star. The rotor winding is assumed open-circuited so that rotor current is zero and no electromagnetic torque is developed. Application of 3- ϕ balanced voltages at line frequency f_1 to the stator winding, causes the production of a rotating magnetic field. The rotating flux cuts both the stator and stationary rotor conductors at synchronous speed, consequently emfs of line frequency f_1 are induced in them. Per phase value of the emf E_1 induced in stator winding is given by

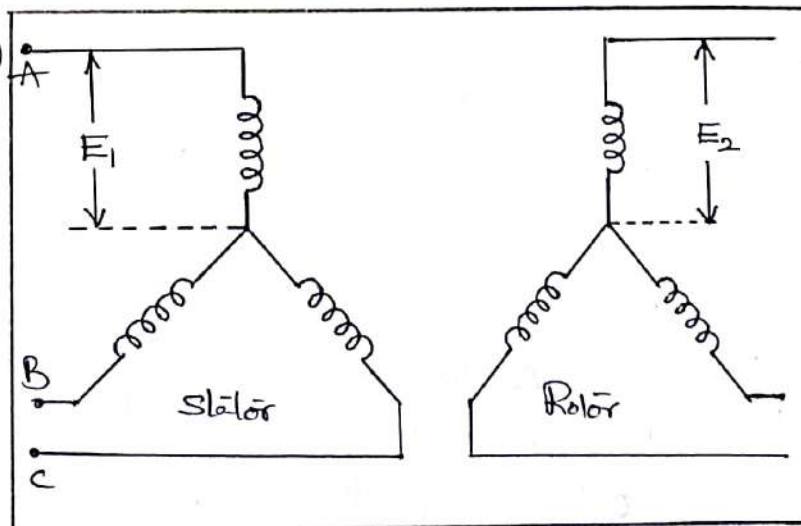
$$E_1 = \sqrt{2} \pi f_1 K_{w1} N_1 \phi \quad \text{--- ①}$$

$$= 4.44 f_1 K_{w1} N_1 \phi$$

Here N_1 = Stator series turns per phase

K_{w1} = Stator winding factor

Similarly per phase value of emf E_2 induced in the standstill rotor winding is given by



$$E_2 = \sqrt{2} \pi f_1 K_{w2} N_2 \phi \quad \text{--- (2)}$$

Here N_2 = Rotor Series turns per phase

K_{w2} = Rotor Winding factor

Line frequency f_1 appears in the expression for E_2 also, because rotor is at standstill. Here ϕ is the resultant air-gap flux per pole.

The emf or voltage ratio for the induction machine from (1) & (2)

$$\frac{E_1}{E_2} = \frac{N_1 K_{w1}}{N_2 K_{w2}} = \frac{N_1'}{N_2'} \quad \text{--- (3)}$$

Here N_1' and N_2' are called the effective number of stator-series turns per phase and rotor-series turns per phase respectively.

E_2 (3) is similar to the voltage ratio of a transformer. Thus, a round rotor induction motor at standstill is similar to a transformer at no-load. In induction machine, the rotating air gap flux generates counter emf E_1 in the stator winding similar to the counter emf induced (by the mutual flux) in the primary winding of the transformer. As the transformer is loaded, the mmf of the secondary current reacts on the primary in order to draw more power from A.C. source. Similarly with the increase in shaft load, the rotor mmf reacts on the stator winding in order to extract more power from the A.C. source. A 3- ϕ induction motor with its rotor blocked behaves similar to a transformer under short circuit. In view of these similarities, an induction machine is called a "generalized transformer" and it is for this reason that the stator winding of an induction machine is treated as primary and rotor winding as its secondary.

The voltage ratio of the induction machine includes winding factor K_{w1} & K_{w2} , because the stator & rotor windings are distributed along the air-gap periphery. The transformer primary & secondary wdggs, being concentrated wdggs, requires no winding factors. In the above eq's (1) & (2) ϕ is the average flux per pole, in transformer emf, maximum value of the core flux is used.

Another difference between induction motors and transformers is that the no-load current in induction motors varies from about 30 to 50% of full-load current. Whereas in transformers, no-load current varies from 2 to 6% of full load current.

This is because of the fact that the mutual flux in a TLF completes its path through low-reluctance iron core, whereas the mutual in an induction motor has to cross the air gap between stator & rotor. For a constant applied voltage, the mutual flux remains substantially constant. Since the reluctance offered to the mutual flux in an induction motor is more due to the presence of air gap, a constant mutual flux (mmf/reluctance) requires more magnetizing mmf and, therefore, more magnetizing current. In induction motors, the magnetizing current (lagging nearly 90° behind the applied voltage) forms a considerable portion of no-load current, that is why induction motors operate at low power factors at no loads. Usually the no-load power factors of induction motors are in the neighbourhood of 0.15 or so.

The effect of low value of no-load power factor is to decrease the full-load operating power factor of induction motor. Thus, for keeping the induction motor magnetizing current low and, therefore, a better no load and full load power factors, air gap in induction motors are kept as small as are mechanically possible.

* Rotor EMF, Current and Power:

At Standstill Rotor E.M.F, $E_2 = \sqrt{2} \pi f_1 K_{w2} N_2 \Phi$.

In Running Condition Rotor E.M.F, $sE_2 = \sqrt{2} \pi f_1 K_{w2} N_2 \Phi \cdot s$

$$= \sqrt{2} \pi (sf_1) K_{w2} N_2 \Phi$$

$$sE_2 = \sqrt{2} \pi f_2 K_{w2} N_2 \Phi.$$

Now rotor leakage Reactance = $2\pi \times \text{Rotor freq} \times \text{Rotor leakage Inductance}$

$$\text{Rotor leakage Reactance at Standstill} = 2\pi f_1 l_2 = x_2 \Omega$$

$$\text{Rotor leakage Reactance in running} = 2\pi f_2 l_2 = 2\pi (sf_1) l_2 = s \cdot x_2 \Omega$$

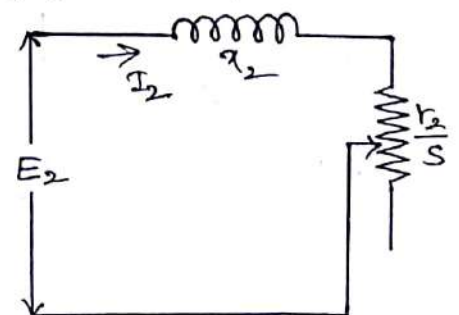
$$\text{Rotor leakage Impedance at standstill} = \sqrt{r_2^2 + x_2^2}$$

$$\text{Rotor leakage Impedance in running} = \sqrt{r_2^2 + (sx_2)^2}$$

per phase rotor current at stand still

$$= \frac{E_2}{\sqrt{r_2^2 + x_2^2}}$$

per phase rotor current at any slip is given by,



$$I_2 = \frac{SE_2}{\sqrt{r_2^2 + (s r_2)^2}} = \frac{E_2}{\sqrt{\left(\frac{r_2}{s}\right)^2 + x_2^2}}$$

The rotor current I_2 lags the rotor voltage E_2 by rotor power factor angle θ_2 given by

$$\theta_2 = \tan^{-1}\left(\frac{S x_2}{r_2}\right)$$

Per phase power input to rotor, $P_2 = E_2 I_2 \cos \theta_2$

$$\cos \theta_2 = \frac{\text{Per phase Rotor resistance}}{\text{per phase Rotor Impedance}} = \frac{r_2/s}{\sqrt{\left(\frac{r_2}{s}\right)^2 + x_2^2}}$$

$$P_2 = E_2 \cdot I_2 \cdot \frac{r_2/s}{\sqrt{\left(\frac{r_2}{s}\right)^2 + x_2^2}}$$

$$P_2 = \frac{E_2}{\sqrt{\left(\frac{r_2}{s}\right)^2 + x_2^2}} I_2 \cdot \frac{r_2}{s} = I_2 \cdot I_2 \cdot \frac{r_2}{s}$$

$$\boxed{P_2 = I_2^2 \frac{r_2}{s}} = \text{Rotor input}$$

$$\therefore P_2 = I_2^2 \frac{r_2}{s} = I_2^2 r_2 + I_2^2 r_2 \left(\frac{1}{s} - 1\right)$$

$$= \left[\text{Rotor ohmic losses} \right] + \left[\text{Internal Mech. Power developed in rotor, } P_m \right]$$

$$P_2 = s P_2 + (1-s) P_2$$

from the above expression

$$P_2 = \text{Rotor input}$$

$$s P_2 = \text{Rotor cu losses}$$

$$P_m = (1-s) P_2 = \text{Mech. power developed in rotor}$$

$$\boxed{P_2 : P_m : \text{Rotor cu losses} = 1 : (1-s) : s}$$

Internal (gross) torque developed per phase is given by

$$T_g = \frac{P_2}{2\pi N_s} = \frac{P_2}{\omega_s} = \frac{P_m}{\omega_r}$$

$$T_g = \frac{P_m}{(1-s)\omega_s} = \text{Electromagnetic torque}$$

$$T_m = \frac{P_2}{\omega_s} = \frac{1}{\omega_s} \cdot \frac{I_2^2 r_2}{s} = \frac{\text{Rotor ohmic loss}}{\omega_s \times \text{slip}}$$

$$= \frac{1}{2\pi n_s} \frac{I_2^2 r_2}{s}$$

output torque / shaft torque, $T_{sh} = \frac{P_{sh}}{\omega_r} = \frac{P_{out}}{\omega_r}$

$$T_{sh} = \frac{P_{out}}{(1-s)\omega_s}$$

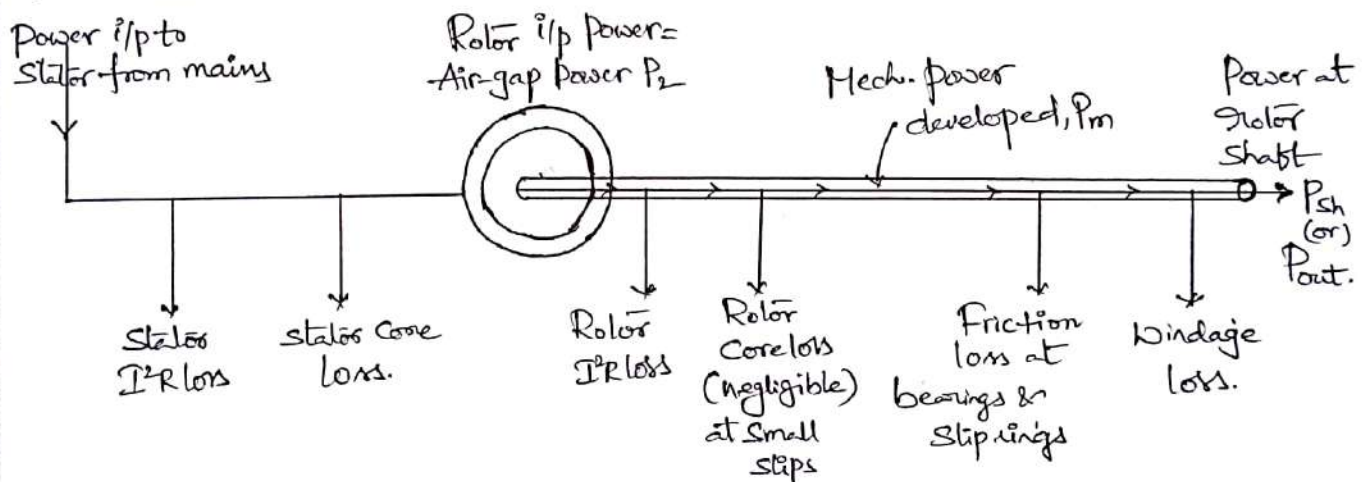
$$P_{sh} = P_{out} = [\text{Stator Power}]_{\text{input}} - [\text{stator loss}] - [\text{Rotor loss}] - [\text{Mech. loss}]$$

$$P_2 = [\text{Stator Power}]_{\text{input}} - [\text{stator core \& cu losses}]$$

Losses & Efficiency:

The losses in a 3- ϕ induction motor are

- (i) Fixed losses [Fixed loss = (Power i/p at no-load) - (stator I²R loss at no-load)]
- (ii) Variable losses.



The efficiency of a 3- ϕ induction motor is given by

$$\eta = \frac{P_{sh}}{P_{sh} + P_f + P_{oh}} \times 100$$

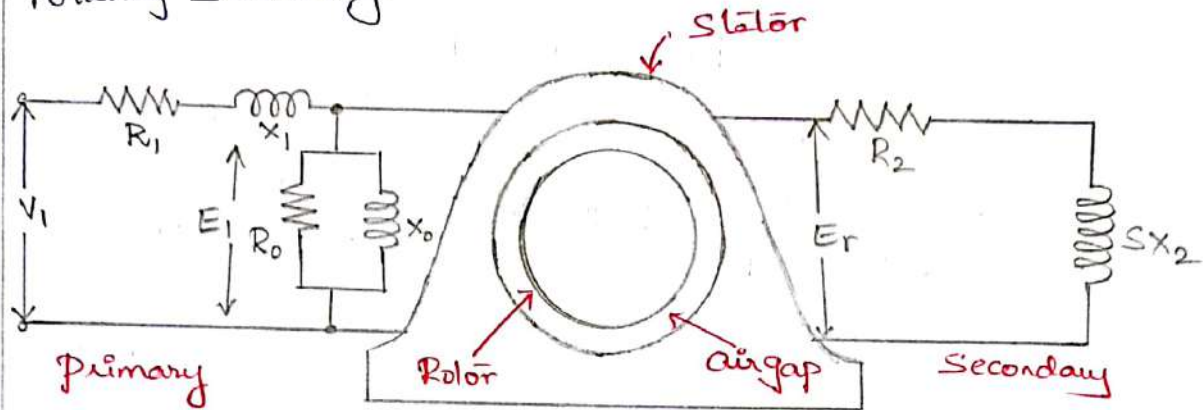
Here, P_{sh} = Shaft Power (or) output power

P_f = fixed losses = core losses + friction & windage losses

P_{oh} = ohmic losses = stator & rotor ohmic losses + brush contact drop (if any)

* Induction Motor as a Generalized Transformer:

The transfer of energy from stator to the rotor of an IM takes place entirely inductively, with the help of a flux mutually linking the two. Hence, an induction motor is essentially a transformer with stator forming the primary and rotor forming rotating secondary.



Where $V_1 \rightarrow$ applied voltage of stator per phase

R_1 & $X_1 \rightarrow$ stator resistance & leakage reactance of stator per phase respectively.

$E_1 \rightarrow$ stator induced emf

$E_r \rightarrow$ rotor induced emf (Running Condition).

$R_2, X_2 \rightarrow$ resistance & leakage reactance of rotor per phase respectively.

From the transformer analysis,

$$V_1 = E_1 + I_1(R_1 + jX_1)$$

$I_1 \rightarrow$ stator current per phase

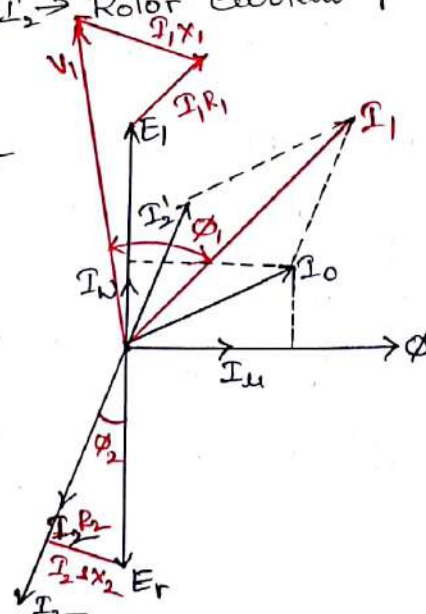
$$\text{and } E_r = I_2 Z_2 = I_2 [R_2 + jSX_2]$$

$I_2 \rightarrow$ rotor current per phase

$I_w \rightarrow$ Working or Iron loss Component

$I_m \rightarrow$ Magnetising Component

$$I_0 = \sqrt{I_w^2 + I_m^2}$$

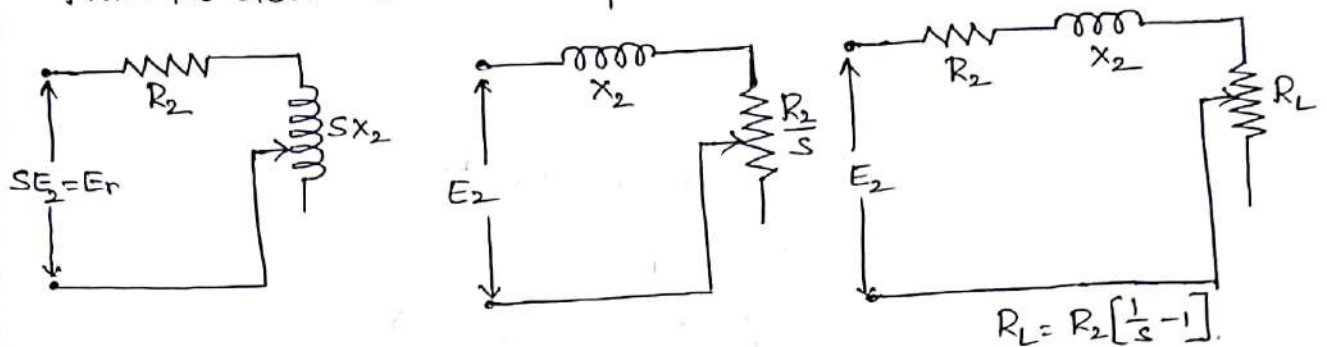


Equivalent Circuit of the Rotor:

When motor is loaded, the rotor current I_2 is given by

$$I_2 = \frac{SE_2}{\sqrt{R_2^2 + (SX_2)^2}} = \frac{E_2}{\sqrt{(R_2/s)^2 + X_2^2}}$$

From the above current expression,

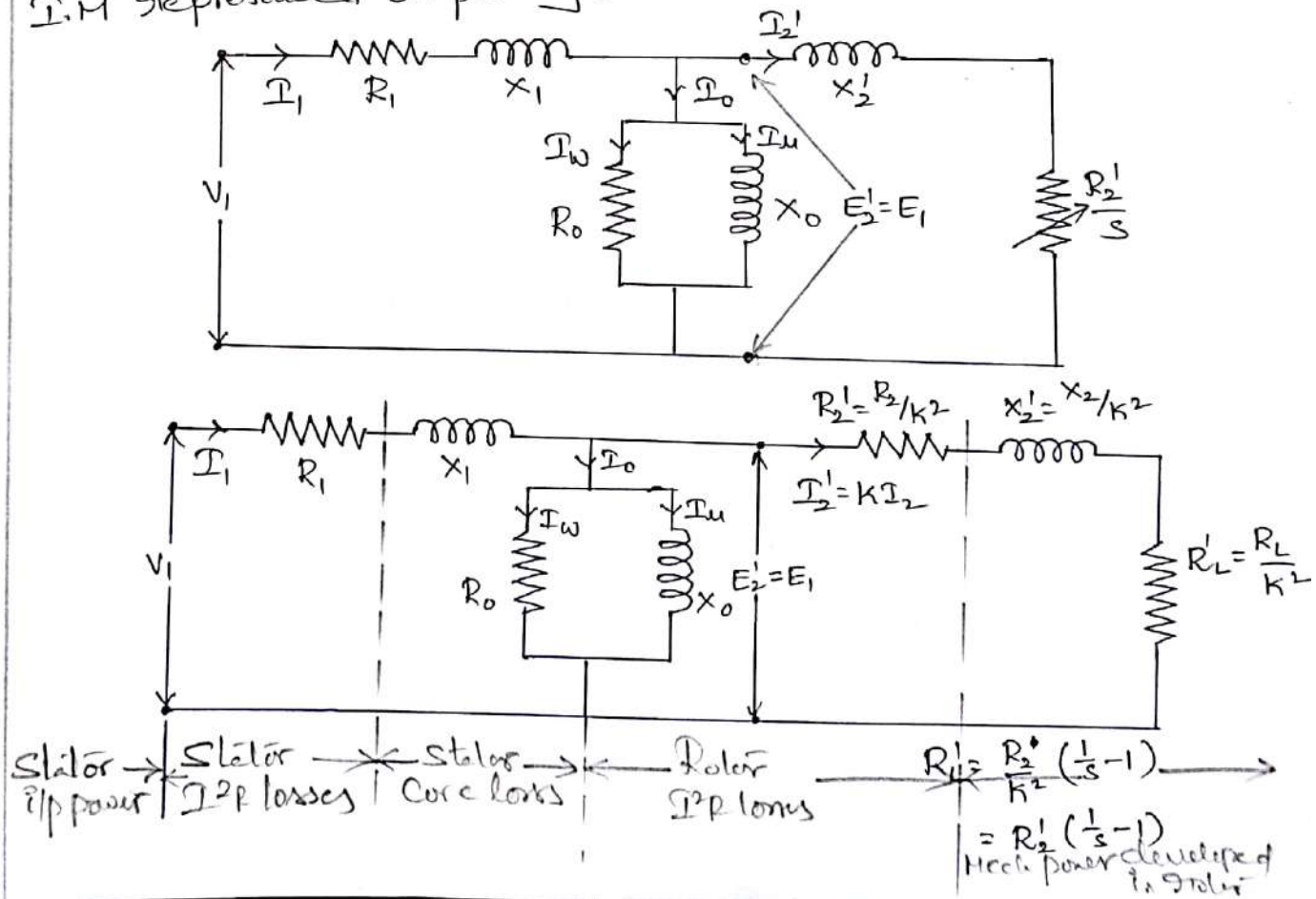


Now, the Resistance $\frac{R_2}{s} = R_2 + R_2\left(\frac{1}{s} - 1\right)$. It consists of two parts:

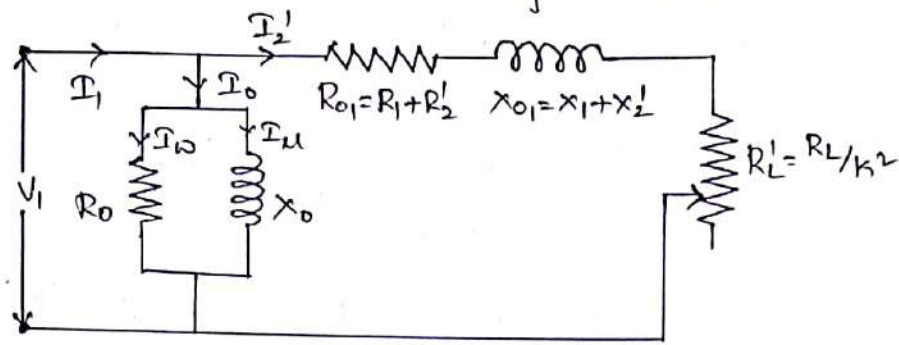
- (i) R_2 is the rotor Resistance itself and represents the rotor cu loss.
- (ii) $R_2\left(\frac{1}{s} - 1\right)$. This is known as load resistance R_L and it is electrically equivalent of the mechanical load on the motor.

* Equivalent Circuit of an Induction Motor:

The Secondary (Rotor) values may be transferred to primary (Stator), from above analysis, the equivalent circuit of a 3- ϕ I.M represented on primary (stator) side as shown below:



Approximate Equivalent Circuit of the induction Motor is



$$I_2' = K \cdot I_2$$

$$R_{01} = R_1 + R_2' = R_1 + \frac{R_2}{K^2}$$

$$X_{01} = X_1 + X_2' = X_1 + \frac{X_2}{K^2}$$

$$R_L' = \frac{R_L}{K^2} = \frac{R_2}{K^2} \left(\frac{1}{s} - 1 \right)$$

* Power Balance Equations:

From the equivalent circuit of an induction motor

$$\text{Input power} = 3V_1 I_1 \cos \phi$$

$$\text{Stator Core loss} = I_w^2 R_0$$

$$\text{Stator cu loss} = 3I_1^2 R_1$$

$$\text{Power transferred to rotor} = \frac{3I_2'^2 R_2'}{s}$$

$$\text{Rotor cu losses} = 3I_2'^2 R_2'$$

Mechanical power developed by motor (P_m) or gross power developed by motor (P_g) = Rotor input - Rotor cu losses

$$= \frac{3I_2'^2 R_2'}{s} - 3I_2'^2 R_2'$$

$$P_g = 3I_2'^2 R_2' \left(\frac{1}{s} - 1 \right)$$

If the T_g is the gross torque developed by the rotor, then

$$T_g \times \omega = T_g \times 2\pi \frac{N}{60} = 3I_2'^2 R_2' \left(\frac{1-s}{s} \right)$$

$$T_g = \frac{3I_2'^2 R_2' \left(\frac{1-s}{s} \right)}{2\pi \frac{N}{60}}$$

$$\text{W.K.T } N = N_s(1-s)$$

$$\therefore T_g = \frac{3I_2'^2 R_2' \left(\frac{1-s}{s} \right)}{\frac{2\pi}{60} N_s(1-s)} = 9.55 \times \frac{3I_2'^2 R_2' / s}{N_s} \text{ N-m}$$

Since gross torque in synchronous watts is equal to the power transferred to the rotor across the air-gap.

$$\therefore T_g = 3 I_2'^2 R_2' / s \quad \text{Sync. Watt}$$

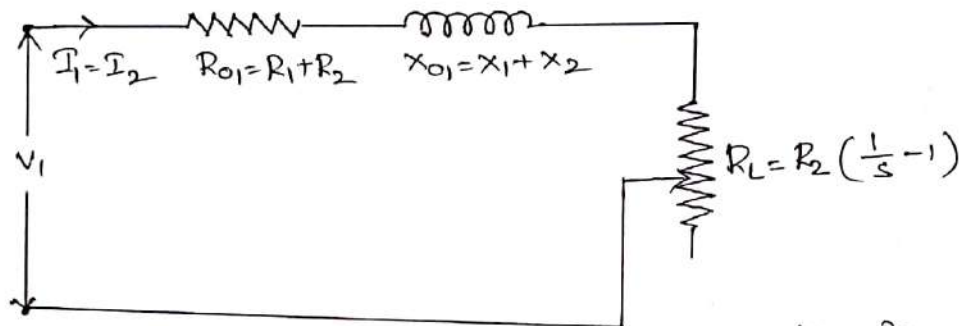
$$I_2' = \frac{V_1}{(R_1 + R_2'/s) + j(X_1 + X_2')}$$

$$T_g = \frac{3}{2\pi N_s/60} \times \frac{V_1^2}{(R_1 + R_2'/s)^2 + (X_1 + X_2')^2} \times \frac{R_2'}{s} \quad \text{N-m}$$

Maximum Power Output:

From the approximate equivalent circuit of an induction motor with the simplification that:

- (i) exciting circuit is omitted i.e., I_0 is neglected and
- (ii) K is assumed unity, then



Gross power output for 3- ϕ induction motor is

$$P_g = 3 I_1^2 R_L$$

$$\text{Now, } I_1 = \frac{V_1}{\sqrt{(R_{01} + R_L)^2 + X_{01}^2}}$$

$$\therefore P_g = \frac{3 V_1^2 R_L}{(R_{01} + R_L)^2 + X_{01}^2}$$

For the maximum power output $\frac{dP_g}{dR_L} = 0$.

$$\frac{dP_g}{dR_L} = \frac{3 V_1^2 R_L [2(R_{01} + R_L) + 0] - [(R_{01} + R_L)^2 + X_{01}^2] 3 V_1^2}{[(R_{01} + R_L)^2 + X_{01}^2]^2} = 0$$

$$2 V_1^2 R_L (R_{01} + R_L) = [(R_{01} + R_L)^2 + X_{01}^2] 3 V_1^2$$

$$2 R_L R_{01} + 2 R_L^2 = R_{01}^2 + R_L^2 + 2 R_{01} R_L + X_{01}^2$$

$$R_L^2 = R_{01}^2 + X_{01}^2$$

$$R_L = Z_{01}$$

Where Z_{01} = Impedance of Induction Motor Refer to stator.
Hence, the power output is maximum "When the equivalent load resistance is equal to the standstill leakage impedance of the motor"

Corresponding slip:

$$\text{Now } R_L = R_2 \left(\frac{1}{s} - 1 \right)$$

$$\therefore Z_{01} = R_L = R_2 \left(\frac{1}{s} - 1 \right) \text{ (or)}$$

$$s = \frac{R_2}{R_2 + Z_{01}}$$

This is the slip corresponding to maximum gross power output. The value of P_{gmax} is obtained by substituting R_L by Z_{01} in the above power equation.

$$P_{gmax} = \frac{3V_1^2 Z_{01}}{(R_{01} + Z_{01})^2 + X_{01}^2} = \frac{3V_1^2 Z_{01}}{R_{01}^2 + Z_{01}^2 + 2R_{01}Z_{01} + X_{01}^2}$$

$$P_{gmax} = \frac{3V_1^2 Z_{01}}{Z_{01}^2 + Z_{01}^2 + 2R_{01}Z_{01}}$$

$$P_{gmax} = \frac{3V_1^2 Z_{01}}{2(Z_{01}^2 + R_{01}Z_{01})}$$

$V_1 \rightarrow$ voltage/phase

$K \rightarrow$ unity

Prob:

A 3- ϕ , star connected 400V, 50Hz, 4-pole Induction motor has the following per phase parameters in ohms, referred to the stator.

$$R_1 = 0.15, X_1 = 0.45, R_2' = 0.12, X_2' = 0.45, X_m = 28.5$$

Compute the stator current & power factor when the motor is operated at rated voltage and frequency with $s = 0.04$.

Sol:

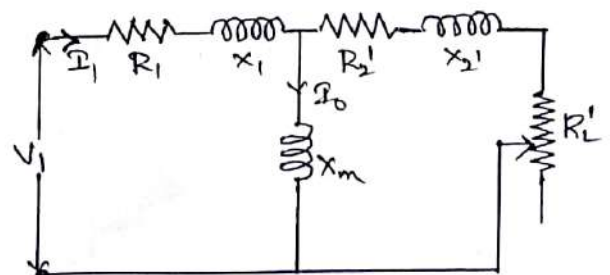
Given that

$$\begin{aligned} R_1 &= 0.15 \Omega \\ X_1 &= 0.45 \Omega \\ R_2' &= 0.12 \Omega \\ X_2' &= 0.45 \Omega \\ X_m &= 28.5 \Omega \end{aligned}$$

$$R_{01} = R_1 + R_2' = 0.15 + 0.12 = 0.27 \Omega$$

$$X_{01} = X_1 + X_2' = 0.45 + 0.45 = 0.9 \Omega$$

$$R_L' = R_2' \left(\frac{1}{s} - 1 \right) = 0.12 \left(\frac{1}{0.04} - 1 \right) = 2.88 \Omega$$



$$I_2' = \frac{V}{(R_{01} + R_L') + jX_{01}} = \frac{400/\sqrt{3}}{(0.27 + 2.88) + j(0.45 + 0.45)} = (67.78 - j19.36) \text{ A}$$

$$I_0 = \frac{V}{jX_m} = \frac{400/\sqrt{3}}{j28.5} = -j8.1$$

$$\text{Stator current, } I_1 = I_0 + I_2' = 73.13 \angle -22^\circ \text{ A}$$

$$\text{Stator power factor, } \cos \phi = \cos 22^\circ = 0.927 (\text{lag})$$

Prob: A 440V, 3- ϕ , 50Hz, 37.3kW, Y-connected induction motor has the following parameters:

$$R_1 = 0.1\Omega, X_1 = 0.4\Omega, R_2' = 0.15\Omega, X_2' = 0.44\Omega$$

Motor has stator core loss of 1250W and rotational loss of 1000W. It draws a no-load line current of 20A at a p.f. of 0.8 (lag). When motor operates at a slip of 3%. Calculate

- (i) input line current and power factor
- (ii) Electromagnetic torque developed in N-m
- (iii) output and
- (iv) efficiency of the motor.

Sol: Line Voltage = 440V, Voltage/ph = $\frac{440}{\sqrt{3}} = 254 \text{ V}$.

$$I_2' = \frac{V}{(R_{01} + R_L') + jX_{01}}$$

$$R_{01} = R_1 + \frac{R_2'}{s} = 0.1 + \frac{0.15}{0.03} = 5.1\Omega$$

$$X_{01} = X_1 + X_2' = 0.4 + 0.44 = 0.84\Omega$$

$$I_2' = \frac{254 \angle 0^\circ}{5.1 + j0.84} = 49.1 \angle -9.3^\circ \text{ A}$$

In no-load condition, the motor draws magnetising current only.

$$I_0 = 20 \angle -84.9^\circ \text{ A}$$

$$(i) \text{ input line current, } I_1 = I_0 + I_2' = 20 \angle -84.9^\circ + 49.1 \angle -9.3^\circ$$

$$I_1 = 57.4 \angle -29^\circ \text{ A}$$

$$\text{Power factor, } \cos \phi = \cos 29^\circ = 0.875 \text{ lagging.}$$

(ii) Electromagnetic torque

$$T_g = 9.55 \times \frac{3 I_2'^2 R_2' / s}{N_s}, \quad N_s = \frac{120f}{P} = \frac{120 \times 50}{4} = 1500 \text{ rpm.}$$

$$T_g = 9.55 \times \frac{3 \times 49.1^2 \times (0.15 / 0.03)}{1500} = 230 \text{ N-m.}$$

(iii) output, $P_m = (1-s)P_2$

$$P_2 = \text{Rotor i/p} = 3 I_2'^2 R_2' / s = 3 \times 49.1^2 \times \frac{0.15}{0.03} = 36,160 \text{ W}$$

$$P_m = (1 - 0.03) \times 36,160 = 35,075 \text{ W. } \times \text{ O/p power} = 35,075 - 1000 = 34,075 \text{ W}$$

(iv) Core loss = 1250W

Stator cu loss = $3I_1^2 R_1 = 3 \times 57.4^2 \times 0.1 = 988W$

Rotor cu loss = $3I_2^2 R_2 = s P_2 = 0.03 \times 36,160 = 1085W$

Rotational losses = 1000W

Total losses = $988 + 1085 + 1000 + 1250 = 4323W$

$\% \eta = \frac{O/p}{O/p + \text{losses}} \times 100 = \frac{34,075}{34,075 + 4,323} = 88.7\%$

Prob: A 40HP 3- ϕ induction motor has a full load slip of 4%. The stator losses amount to 4% of the input of the motor and the mechanical losses are 1% of the output. If the current in each phase of the motor is 50A. Find the resistance per phase of the motor and the efficiency of this machine. (April/May-2017, Set-1)

Sol: Given that Output, $P_{out} = 40HP = 40 \times 746W = 29840W$

full load slip, $s = 4\% = 0.04$

stator losses, $P_{st} = 4\%$ of the input

Mechanical losses, $P_{mech} = 1\%$ of the output
 $= \frac{1}{100} \times 29840 = 298.4W$

Rotor current/ph $= I_2 = 50A$
 From the eqⁿ $P_2 : P_m : \text{Rotor cu loss} = 1 : (1-s) : s$

$P_m = \text{Mech. power developed in rotor} = \text{Rotor o/p} + \text{Mech. losses}$
 $= 29840 + 298.4$
 $= 30138.4W$

(i) $\frac{P_m}{\text{Rotor cu loss}} = \frac{1-s}{s} \Rightarrow \frac{P_m}{\text{Rotor cu loss}} = \frac{1-0.04}{0.04} = 24$

Rotor cu loss = 1255.8W

$3I_2^2 R_2 = 1255.8$

$R_2 = \frac{1255.8}{3I_2^2} = \frac{1255.8}{3 \times 50^2}$

Rotor Resistance, $R_2 = 0.167 \Omega / \text{ph}$

(ii) $\frac{P_2}{\text{Rotor cu loss}} = \frac{1}{s} \Rightarrow \frac{P_2}{1255.8} = \frac{1}{0.04}$

Rotor input, $P_2 = \frac{1255.8}{0.04} = 31394W$

Stator losses = stator i/p - Rotor i/p

Stator i/p = Rotor i/p + stator cu losses.

$$P_1 = P_2 + P_{st}$$

$$P_1 = P_2 + 4\% \text{ of } P_1$$

$$P_1 = P_2 + \frac{4}{100} P_1 \Rightarrow P_1 - 0.04 P_1 = P_2$$

$$P_1 = \frac{P_2}{0.96} = \frac{31394}{0.96}$$

$$\text{Stator i/p, } P_1 = 32702.08 \text{ W.}$$

$$\text{Stator losses, } P_{st} = 4\% \text{ of } P_1 = 1308.08 \text{ W.}$$

$$\text{Motor efficiency, } \eta = \frac{P_{out}}{P_1} \times 100 = \frac{P_1}{P_1 - \text{losses}} \times 100$$

$$\% \eta = \frac{29840}{32054.7} \times 100 = 91.2\%$$

$$\boxed{\% \eta = 91.2\%}$$

Prob: A 3- ϕ , 50Hz induction motor has a full-load speed of 1440 rpm. For this motor, calculate the following:
 (i) Number of poles (ii) Full load slip & rotor frequency (iii) Speed of stator field with respect to A) stator structure & B) also rotor structure
 (iv) Speed of rotor field with respect to A) rotor structure and B) stator field. (April/May-2017, set-2)

Sol: Given that, full load speed, $N = 1440 \text{ rpm}$.
 Synchronous speed, $N_s = 1500 \text{ rpm}$ (Nearest to N)

$$(i) \text{ No. of poles, } P = \frac{120f}{N_s} = \frac{120 \times 50}{1500} = 4.$$

$$(ii) \text{ Full load slip, } S = \frac{N_s - N}{N_s} = \frac{1500 - 1440}{1500} = 0.04 = 4\%.$$

$$\text{Rotor frequency, } f_r = Sf = 0.04 \times 50 = 2 \text{ Hz.}$$

(iii) Speed of stator field with respect to

A) stator structure, $N_s = 1500 \text{ rpm}$.

B) Rotor structure, ^{slip} speed = 60 rpm.

(iv) Speed of rotor field with respect to

A) Rotor structure, slip speed = $\frac{120f_r}{P} = \frac{120 \times 2}{4} = 60 \text{ rpm}$.

B) stator field, zero.

C) stator structure, $N_s = 1500 \text{ rpm}$.

Prob: A 3- ϕ , 400V, 50Hz induction motor takes a power input of 35kW at its full load speed of 980 rpm. The total stator losses are 1kW and friction & windage losses are 1.5kW. Calculate (i) slip (ii) motor ohmic losses (iii) shaft power (iv) shaft torque (v) efficiency.
[April/May-2017, Set-3]

Sol: Given that Voltage, $V = 400V$, $f = 50Hz$
Power input, $P_1 = 35kW$
full load speed, $N = 980 \text{ rpm}$
stator losses, $P_{st} = 1kW$
Mech. losses, $P_{mech} = 1.5kW$

(i) Synchronous Speed, $N_s = 1000 \text{ rpm}$ (Nearest to N)

$$\% \text{ slip} = \frac{N_s - N}{N_s} \times 100 = \frac{1000 - 980}{1000} \times 100 = 2\%$$

$$s = 0.02$$

(ii) Rotor ohmic losses:

$$P_2 : P_m : \text{Rotor cu losses} = 1 : (1-s) : s$$

$$\text{Rotor e/p, } P_2 = P_1 - \text{stator losses} = 35K - 1K = 34KW.$$

$$\frac{P_2}{\text{Rotor cu losses}} = \frac{1}{s} \Rightarrow \frac{34K}{\text{Rotor cu losses}} = \frac{1}{0.02}$$

$$\text{Rotor cu losses} = 680W$$

(iii) Shaft power / Mech. power developed in rotor, P_m

$$\frac{P_2}{P_m} = \frac{1}{1-s} \Rightarrow \frac{34K}{P_m} = \frac{1}{1-0.02}$$

$$\frac{34K}{P_m} = \frac{1}{0.98} \Rightarrow P_m = 34K \times 0.98$$

$$P_m = 33.32KW$$

$$P_{out} = 31.82KW$$

(iv) Shaft torque, T_{sh}

$$T_{gross} = \frac{P_m}{2\pi N_s} = \frac{33.32K}{2 \times 3.14 \times \frac{1000}{60}} = 318.34 \text{ N-m.}$$

$$T_{sh} = \frac{P_{out}}{2\pi N}$$

$$P_{out} = P_m - P_{mech} = 33.32K - 1.5K = 31.82KW$$

$$T_{sh} = \frac{31.82K}{2 \times 3.14 \times \frac{980}{60}} \Rightarrow$$

$$T_{sh} = 310.2 \text{ N-m}$$

Prob: A 3- ϕ , 4-pole, 400V, 50Hz induction motor develops 15 bhp at 1425 rpm with a power factor of 0.8. The mechanical losses amount to 0.75 hp. Determine for this load.
 (i) slip (ii) motor losses (iii) gross torque developed in synchronous watt (iv) line current. Take the stator losses equal to 1200W. (April/May-2017, Set-3)

Prob: A 20hp, 400V, 50Hz, 3- ϕ induction motor runs at 970 rpm at full load. The mechanical losses are 600W. If the stator losses are 750W find motor cu losses and efficiency of the motor at full load.
 (-April/May-2017, Set-4)

* Synchronous Watt:

Synchronous watt is that torque which, at the synchronous speed of the machine under consideration, would develop a power of 1 watt.

$$\text{Motor i/p} = T_{sw} \times 2\pi N_s$$

$$\therefore T_{sw} = \frac{\text{Motor i/p}, P_2}{2\pi N_s}$$

$$= \frac{1}{\omega_s} \times P_2 = \frac{1}{\omega_s} \times \frac{N_s}{N} \times P_g = \frac{1}{\omega_s} \times \frac{N_s}{N} \times P_m$$

Synchronous wattage of an induction motor equals the power transferred across the air gap to the motor.

$$\text{Torque in Sync-watt} = \text{Rotor i/p} = \frac{\text{Rotor cu losses}}{s} = \frac{\text{Gross o/p Power}, P_m}{(1-s)}$$

Prob: A 10KW, 400V, 3- ϕ , 4-pole, 50Hz delta connected induction motor is running at no-load with a line current of 8A and an input power of 660W. At full load, line current is 18A and input power is 11.20KW. Stator effective resistance per phase is 1.2 Ω and friction, windage loss is 420W. For negligible rotor ohmic losses at no load, calculate (i) stator core loss (ii) total rotor losses at full load (iii) total ohmic (rotor) losses at full load (iv) full load speed. (April/May-2016, Set-2)

Sol: Given that Output power, $P_{out} = 10\text{KW}$
 Poles, $P = 4$

Frequency, $f = 50\text{Hz}$

Voltage, $V = 400\text{V}$, Delta Connected

No-load line current, $= 8\text{A}$

No-load i/p power, $= 660\text{W}$

Full-load line current, $I_1 = 18\text{A}$

Full-load i/p power, $P_1 = 11.20\text{KW}$

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Stator effective resistance, $R_1 = 1.2 \Omega$

Friction, windage losses, $P_{mech} = 420 \text{ W}$

Negligible rotor ohmic losses at no-load.

(i) Stator Core losses:

Total losses at No-load = $I_1 P$ Power at No-load

= Stator $I_1 P$ = Stator total losses

(Rotor is o/c $\times 3 I_2^2 R_2$ is negligible)

$\therefore$ No-load $I_1 P$ power = Stator $I_1 P$ = Stator losses

= { Stator cu losses at No-load } + { Stator Core loss }

$$660 \text{ W} = 3 \times I_a^2 \times R_{01} + \text{Stator Core loss} + P_{mech}$$

$$660 = 3 \times \left(\frac{8}{\sqrt{3}}\right)^2 \times 1.2 + \text{Stator Core loss} + P_{mech}$$

$$\boxed{\text{Stator Core loss} = 163.2 \text{ W}}$$

(ii) Total Rotor losses at full load:

Rotor $I_1 P$ = Stator $I_1 P$ at full load - Stator losses at full load

Rotor $I_1 P$ = { Stator $I_1 P$ at full load } - { Stator cu loss at full load + Stator Core loss }

$$\begin{aligned} \text{Rotor } I_1 P &= 11.2 \text{ K} - [3 I_1^2 R_1 + 163.2] \\ &= 11.2 \text{ K} - \left(3 \times \left(\frac{18}{\sqrt{3}}\right)^2 \times 1.2 + 163.2\right) \end{aligned}$$

$$\boxed{P_2 = 10,648 \text{ W}}$$

{ Total Rotor losses at full load } = { Rotor $I_1 P$ } - { Mech. Power developed in Rotor, P_m }

$$\begin{aligned} \text{Mech. power developed in Rotor, } P_m &= \text{Rotor o/p} - \text{Mech. losses} \\ &= 10 \text{ K} - 420 = 9580 \text{ W} \end{aligned}$$

$$\text{Rotor losses} = P_2 - P_{out}$$

$$= 10648 - 10000 = 648 \text{ W}$$

$$\boxed{648 \text{ W} = \text{Total Rotor losses}}$$

(iii) Total Rotor ohmic losses at full load = $3 I_2^2 R_2 =$

[Total Rotor losses - Mech losses]

$$3 I_2^2 R_2 = 648 - 420 = 228 \text{ W.}$$

(iv) Full load slip:

$$\text{W.K.T. } P_2 : P_m : 3 I_2^2 R_2 = 1 : (1-s) : s$$

from the above equation, $S = \frac{s I_2^2 R_2}{P_2} = \frac{22.8}{10648} = 0.0212$

slip at full load, $S = 2.12\%$

Full load speed, $N_r = N_s(1-S) = 1500(1-0.0212)$

$N_r = 1468 \text{ rpm.}$

(v) Internal torque (or) Torque developed in motor

$$T_g = \frac{P_2}{\omega_s} = \frac{10648}{2\pi \times \frac{1500}{60}} = 67.79 \text{ N-m}$$

$T_g = 67.79 \text{ N-m}$

Shaft torque, $T_{sh} = \frac{P_{out}}{2\pi N_r} = \frac{P_{out}}{\omega_r}$

$$= \frac{10000}{2\pi \times \frac{1468}{60}} = 65.04 \text{ N-m}$$

$T_{sh} = 65.04 \text{ N-m}$

Efficiency, $\eta = \frac{P_{out}}{P_{in}} \times 100 = \frac{10000}{11200} \times 100 = 0.892$

$\% \eta = 89.2\%$

* Torque Equation:

From the power flow diagram of 3- ϕ Induction motor, the Internal (gross) torque developed per phase is given by

$$T_e \text{ (or) } T_g = \frac{P_2}{\omega_s}$$

for 3- ϕ , The torque developed in motor is

$$T_e \text{ (or) } T_g = \frac{3}{\omega_s} \cdot P_g$$

$$P_g = P_2 = \text{Rotor input in Running} = I_2^2 \frac{R_2}{s}$$

$$\text{and } I_2 = \frac{E_2}{\sqrt{(R_2/s)^2 + X_2^2}}$$

$$\therefore T_e \text{ (or) } T_g = \frac{3}{\omega_s} \cdot \frac{E_2^2}{(R_2/s)^2 + X_2^2} \cdot \frac{R_2}{s}$$

$$T_g = \frac{3}{2\pi N_s} \cdot \frac{SE_2^2}{r_2^2 + (sX_2)^2} \cdot r_2$$

Torque at Running Conditions:

gross torque developed in Rotor at Running Condition

$$T_g = \frac{3}{2\pi N_s} \cdot \frac{SE_2^2 r_2}{r_2^2 + (sX_2)^2} ; N_s \text{ in rps}$$

Torque at Standstill:

In Standstill Condition Slip $s=1$ ($N_r=0, s=\frac{N_s-0}{N_s}=1$).

gross torque developed in Rotor in standstill condition is

$$T_{gsl} = \frac{3}{2\pi N_s} \cdot \frac{E_2^2 r_2}{r_2^2 + X_2^2} ; N_s \text{ in rps.}$$

Condition for Maximum Torque:

For the Condition of Maximum Torque, $\frac{dT_g}{dr_2} = 0$.

$$\frac{dT_g}{dr_2} = \frac{d}{dr_2} \left[\frac{3}{2\pi N_s} \cdot \frac{SE_2^2 r_2}{r_2^2 + (sX_2)^2} \right]$$

$$= \frac{3}{2\pi N_s} \cdot \frac{SE_2^2 r_2 [2r_2] - (r_2^2 + (sX_2)^2)(SE_2^2)}{[r_2^2 + (sX_2)^2]^2} \quad \text{--- (i)}$$

For Maximum torque Condition $\frac{dT_g}{dr_2} = 0$.

from eqn ①

$$\frac{dT_g}{dr_2} = 0 \Rightarrow \frac{SE_2^2 r_2 (2r_2) - [r_2^2 + (s\alpha_2)^2] \left[\frac{SE_2^2}{2} \right]}{r_2^4} = 0$$

$$2r_2^2 - r_2^2 - (s\alpha_2)^2 = 0$$

$$r_2^2 = (s\alpha_2)^2$$

$$r_2 = s\alpha_2$$

Here s_{MT} = Slip at Maximum Torque

$$\therefore s_{MT} = \frac{r_2}{\alpha_2} \quad \left[\text{Sometimes } s_{MT} \text{ is called '}\alpha\text{' } \right]$$

Now, the maximum torque from eqn ①

$$T_{max} = T_{g,max} = \frac{3}{2\pi N_s} \cdot \frac{SE_2^2 (s\alpha_2)}{(s\alpha_2)^2 + (s\alpha_2)^2}$$

$$T_{g,max} = \frac{3}{2\pi N_s} \cdot \frac{s^2 E_2^2 \alpha_2}{2 s^2 \alpha_2^2}$$

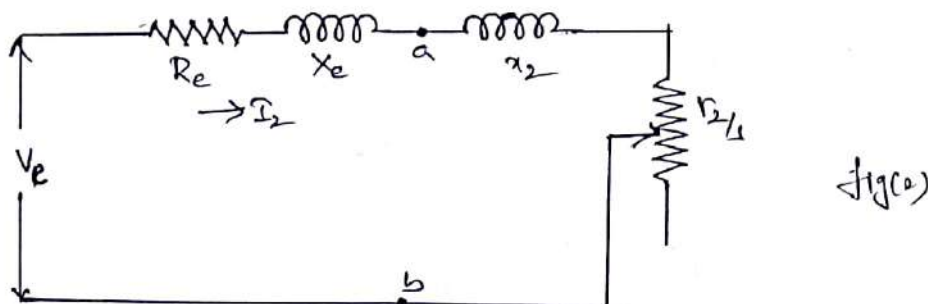
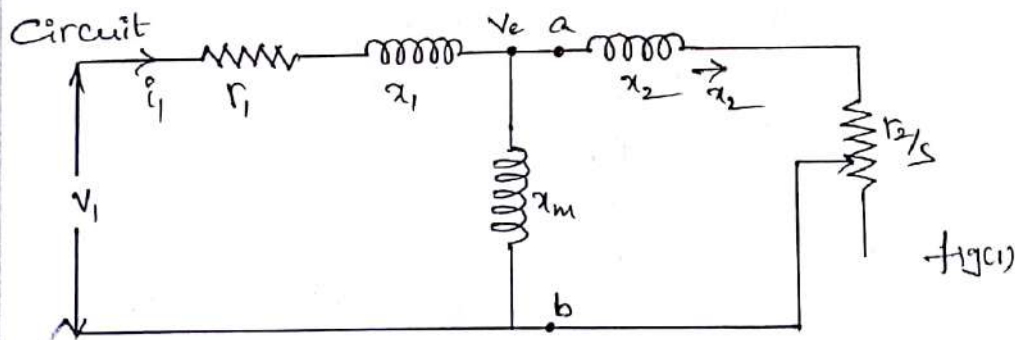
$$T_{g,max} = \frac{3}{\omega_s} \cdot \frac{E_2^2}{2\alpha_2} \quad \text{--- ②}$$

From the above eqn

- (i) The magnitude of Max. Torque independent of Rotor Resistance.
- (ii) But Slip at Maximum Torque depends on Rotor resistance - ($s_{MT} = r_2/\alpha_2$).
- (iii) Max. Torque directly proportional to Square of the Rotor Voltage.

* Torque-Slip Characteristics:

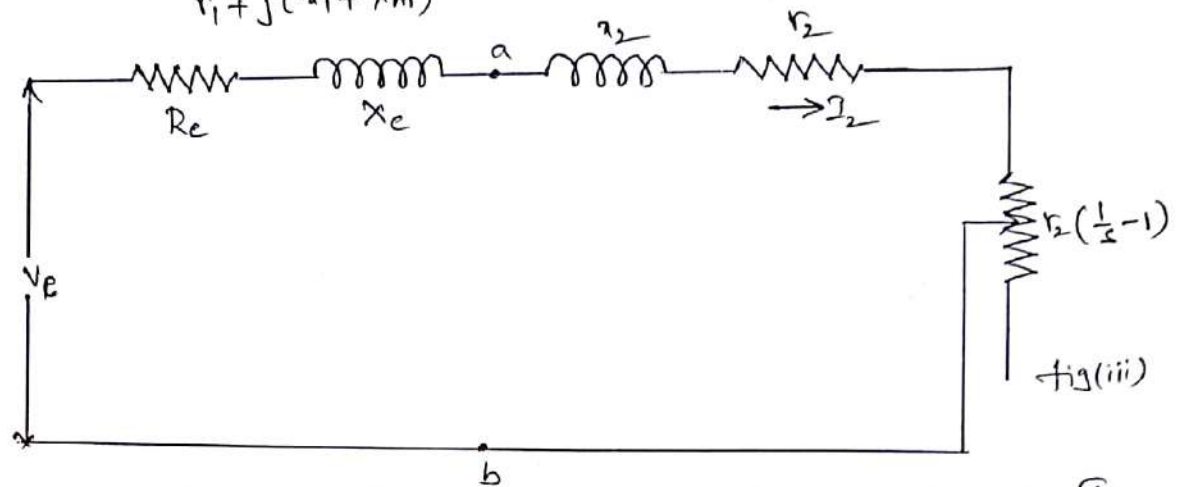
When torque-slip (or) power-slip characteristics are required, application of Thevenin's Theorem to the induction motor equivalent



From the fig (i).

$$V_e = \frac{V_1(jX_m)}{r_1 + jx_1 + jX_m} = \frac{V_1 jX_m}{r_1 + j(x_1 + X_m)}$$

$$Z_e = \frac{(r_1 + jx_1)(jX_m)}{r_1 + j(x_1 + X_m)}$$



For most induction motors, $(x_1 + X_m)$ is much greater than r_1 . In view of this, r_1 can be neglected.

$$\text{Thus, } \bar{V}_e = \frac{V_1 X_m}{x_1 + X_m} = \frac{V_1 X_m}{X_1} \quad (\because x_1 + X_m = X_1)$$

$$\bar{Z}_e = R_e + jX_e = \frac{j^2 r_1 X_m + j^2 x_1 X_m}{j(x_1 + X_m)} \quad (r_1 \rightarrow \text{neglected})$$

$$= \frac{r_1 X_m}{x_1 + X_m} + j \frac{x_1 X_m}{x_1 + X_m}$$

Here $X_1 = x_1 + X_m$ is the stator self-reactance per phase.

From fig (ii)

$$\bar{I}_2 = \frac{\bar{V}_e}{(R_e + \frac{r_2}{s}) + j(X_e + x_2)}$$

Total torque is

$$T_e = T_g = \frac{3}{2\pi N_s} \cdot \frac{\bar{V}_e^2}{(R_e + \frac{r_2}{s})^2 + (X_e + x_2)^2} \cdot \frac{r_2}{s} \quad \text{N-m} \quad \text{---} \textcircled{1}$$

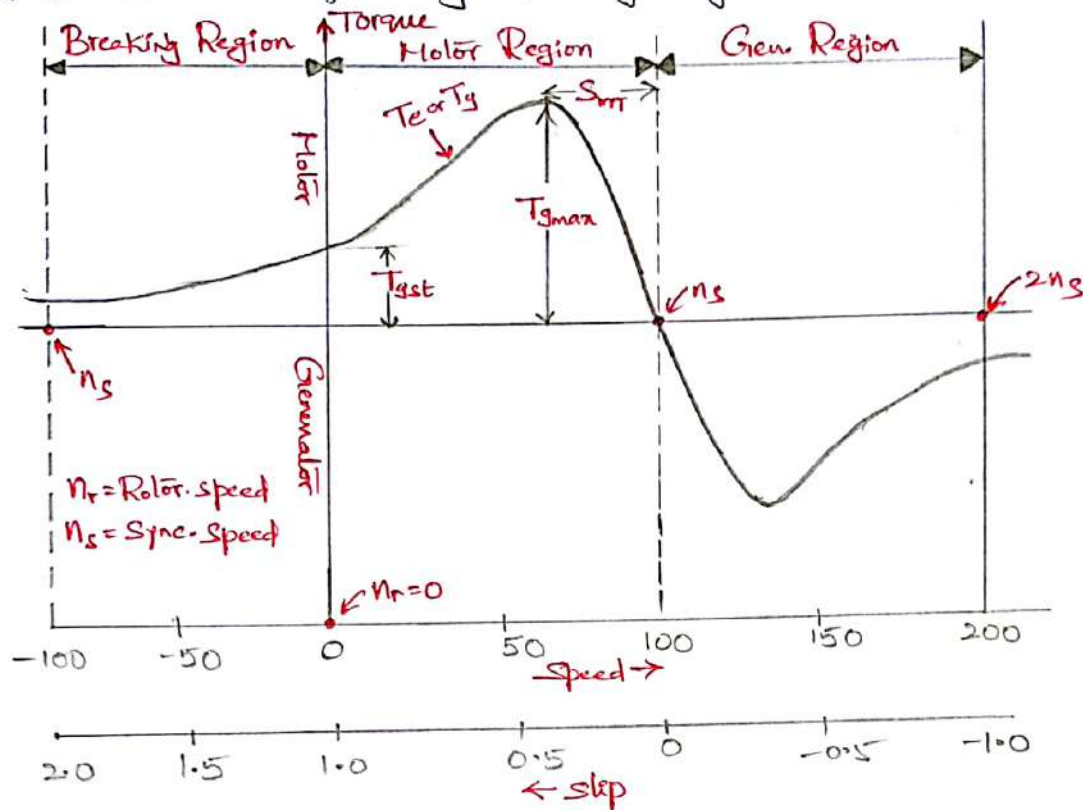
$$T_e = T_g = \frac{K_t}{(R_e + \frac{r_2}{s})^2 + X^2} \cdot \frac{r_2}{s} \quad \text{N-m}$$

$$\text{Here } K_t = \frac{3}{2\pi N_s} \bar{V}_e^2 \quad \text{and } X = x_2 + X_e$$

The variation of torque with slip, or speed, of an induction motor can be plotted from eqn (1) for different values of slip 's' and with the

motor connected to constant-frequency voltage source.

A general shape of the torque-speed (or) torque-slip curve is shown in below figure. Depending on the value of slip an induction motor can have the following operating regions or modes:



(a) Motoring Mode: $0 < S \leq 1$

Under normal operation, rotor revolves in the direction of rotating field produced by the stator currents. As such, the slip varies from 1 to stand still to zero at synchronous speed, i.e., $1 \leq S \leq 0$. The corresponding speed values are zero ($S=1.0$) and synchronous speed ($S=0.0$).

(b) Generating Mode: $S < 0$

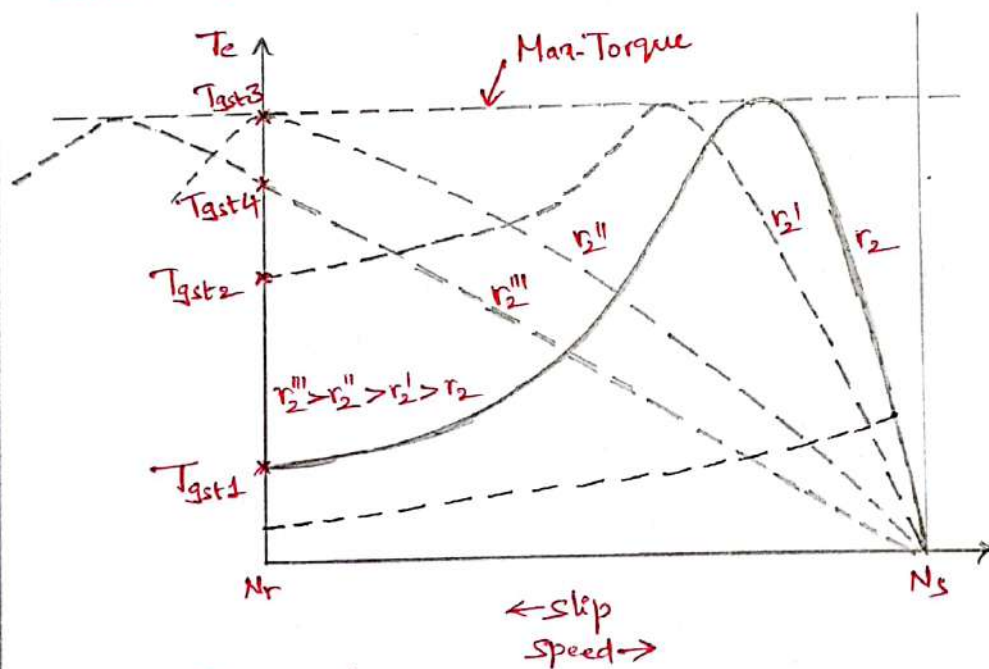
For this operating mode, slip is negative, i.e., $S < 0$. An induction motor will operate in this region only when its stator terminals are connected to constant-frequency voltage source and its rotor is driven above synchronous speed by a prime mover. The connection of stator terminals to voltage source is essential in order to establish the rotating air gap field at synchronous speed. In case stator is disconnected from voltage source and rotor is driven above synchronous speed by the prime mover, no generating action would take place.

(C) Braking Mode: ($s \geq 1$)

For this mode, slip is greater than 1. A slip more than one can be obtained by driving the rotor, with a prime mover, opposite to the direction of rotating field. But such an use in practice is rare. A practical utility of slip more than 1 is obtained by braking the rotor to a quick stop by braking action, called "plugging".

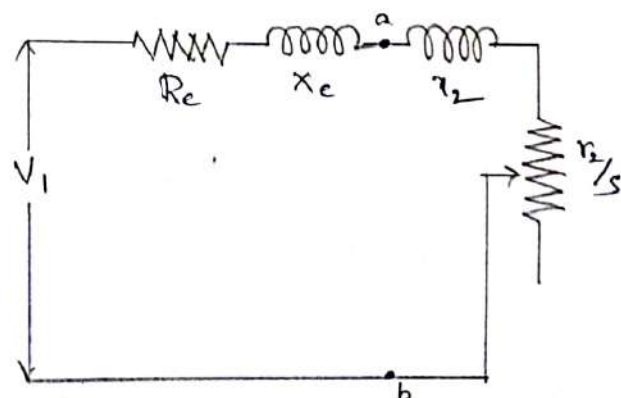
For obtaining $s > 1$, or for obtaining plugging, any two stator leads are interchanged. With this the phase sequence is reversed and, therefore the direction of rotating magnetic field becomes suddenly - opposite to that of the rotor rotation. The electromagnetic torque T_e or T_g , now acting opposite to rotor rotation, produces the braking action. Thus the motor can be quickly brought to rest by plugging; but the stator must be disconnected from the supply before the rotor can start in the other direction.

Induction Motor Torque-slip curves with different values of rotor-circuit Resistance:



Maximum Internal/gross Torque:

This is also referred to as stalling torque, pull-out torque or breakdown torque T_{em} or T_{gm} . The condition for maximum internal torque can be obtained by using Maximum Power Transfer Theorem.



According to Maximum power Transfer theorem, when the power in motor is maximum, the torque developed in motor also maximum (Torque $\propto$ Power developed in rotor), i.e., when Current $I_2^2 r_2 / s$ is maximum, torque is maximum in motor.

For the above equivalent induction motor circuit, apply - Maximum power transfer theorem,

$$\frac{r_2}{S_{MT}} = \sqrt{R_e^2 + (X_e + x_2)^2} = \sqrt{R_e^2 + X^2}$$

Here S_{MT} = Slip at Maximum Torque.

$$\therefore S_{MT} = \frac{r_2}{\sqrt{R_e^2 + X^2}}$$

**/ In some cases R_e & X_e neglected, then $X = x_2$

$$\therefore S_{MT} = \frac{r_2}{\sqrt{x_2^2}} = \frac{r_2}{x_2} = a. \quad /**$$

$$\text{W.K.T } T_e = T_g = \frac{K_t \cdot r_2 / s}{\left(R_e + \frac{r_2}{s}\right)^2 + (X_e + x_2)^2} = \frac{K_t \cdot r_2 / s}{\left(R_e + \frac{r_2}{s}\right)^2 + X^2} ; K_t = \frac{3}{\omega_s} \cdot E_2^2 = \frac{3}{\omega_s} \cdot V_e^2$$

$$\text{Substitute } r_2 / S_{MT} = \sqrt{R_e^2 + X^2}$$

$$T_{em} = T_{gm} = \frac{K_t \cdot r_2 / S_{MT}}{\left(R_e + \frac{r_2}{S_{MT}}\right)^2 + X^2} = \frac{K_t \cdot (\sqrt{R_e^2 + X^2})}{\left(R_e + \sqrt{R_e^2 + X^2}\right)^2 + X^2}$$

$$= \frac{K_t (\sqrt{R_e^2 + X^2})}{R_e^2 + (R_e^2 + X^2) + 2R_e \sqrt{R_e^2 + X^2} + X^2}$$

$$= \frac{K_t \cdot \sqrt{R_e^2 + X^2}}{2R_e^2 + 2X^2 + 2R_e \sqrt{R_e^2 + X^2}} = \frac{K_t \sqrt{R_e^2 + X^2}}{2\sqrt{R_e^2 + X^2} [R_e + \sqrt{R_e^2 + X^2}]}$$

$$\therefore T_{em} = T_{gm} = \frac{K_t}{2[R_e + \sqrt{R_e^2 + X^2}]}.$$

$$**/ \text{ at } S_{MT} = a = \frac{r_2}{x_2}, T_{em} = T_{gm} = \frac{K_t}{2[0 + \sqrt{0 + x_2^2}]} = \frac{K_t}{2x_2}$$

$$T_{em} = T_{gm} = \frac{K_t}{2x_2} \quad /**$$

From the above Maximum Torque Equation, T_{em} or T_{gm}

- (i) T_{em} is directly proportional to the Square of the Voltage.
- (ii) T_{em} is reduced by an increase in stator Resistance r_1 (i.e. R_e) &
- (iii) T_{em} is reduced by an increase in stator leakage reactance x_1 and rotor leakage reactance x_2 .

For obtaining higher value of maximum torque, the airgap is kept as small as is possible. A small air gap allows more flux to be mutual between stator and rotor windings. As consequence, leakage fluxes and therefore leakage reactances are reduced and the magnitude of Maximum torque becomes more.

* Torque Relations:

Torque Relation between Max. Torque & Full load Torque:

Full Load Torque in terms of Maximum Torque:

W.K.T Full load Torque, $T_g = T_e = \frac{K_t \cdot \frac{V_2}{s}}{\left(\frac{r_2}{s}\right)^2 + x_2^2}$ — (1)

W.K.T Maximum Torque, $T_{gmax} = \frac{K_t}{2x_2}$ — (2) [R_e, x_e are neglected].

$$\begin{aligned} \frac{(1)}{(2)} &= \frac{T_g}{T_{gmax}} = \frac{K_t \frac{r_2}{s}}{\left(\frac{r_2}{s}\right)^2 + x_2^2} \cdot \frac{2x_2}{K_t} \\ &= \frac{2x_2 r_2 / s}{\left(\frac{r_2}{s}\right)^2 + x_2^2} = \frac{2 \frac{r_2}{x_2} \frac{1}{s}}{\left(\frac{r_2}{x_2} \frac{1}{s}\right)^2 + 1} \\ &= \frac{2 \frac{S_{MT}}{S}}{\left(\frac{S_{MT}}{S}\right)^2 + 1} = \frac{2 S S_{MT}}{S^2 + S_{MT}^2} \end{aligned}$$

$$T_g = T_{gmax} \frac{2 S S_{MT}}{S^2 + S_{MT}^2}$$

$S \rightarrow$ slip at full load Torque
 $S_{MT} \rightarrow$ slip at Max Torque $= \frac{r_2}{x_2}$

Torque Relation between Max. Torque & Starting Torque:

Starting Torque in terms of Maximum Torque:

W.K.T Starting Torque, $T_{st} = \frac{K_t r_2}{r_2^2 + x_2^2}$ — (1)

W.K.T Maximum Torque, $T_{gmax} = \frac{K_t}{2x_2}$ — (2)

from ① & ②

$$\frac{T_{st}}{T_{gmax}} = \frac{k_t r_2}{r_2^2 + x_2^2} \cdot \frac{2x_2}{k_t} = \frac{2r_2 x_2}{r_2^2 + x_2^2}$$

$$= \frac{2r_2/x_2}{(r_2/x_2)^2 + 1} = \frac{2S_{MT}}{S_{MT}^2 + 1}$$

$$T_{st} = T_{gmax} \frac{2S_{MT}}{S_{MT}^2 + 1}$$

$S_{MT} \rightarrow$ slip at Max. Torque $= \frac{r_2}{x_2}$

* Power-slip Characteristics:

The total internal Mechanical power developed is

$$P_m = m \left(\frac{1-s}{s} \right) I_2^2 r_2 \quad \text{Watt} \quad m \rightarrow \text{No. of stator phases.}$$

$$P_m = m \frac{V_e^2}{\left[R_e + r_2 + \frac{r_2(1-s)}{s} \right]^2 + x^2} r_2 \cdot \left(\frac{1-s}{s} \right)$$

According to maximum power transfer theorem, P_m per phase is the power delivered to $r_2 \left(\frac{1-s}{s} \right)$, internal mechanical power developed is maximum, when

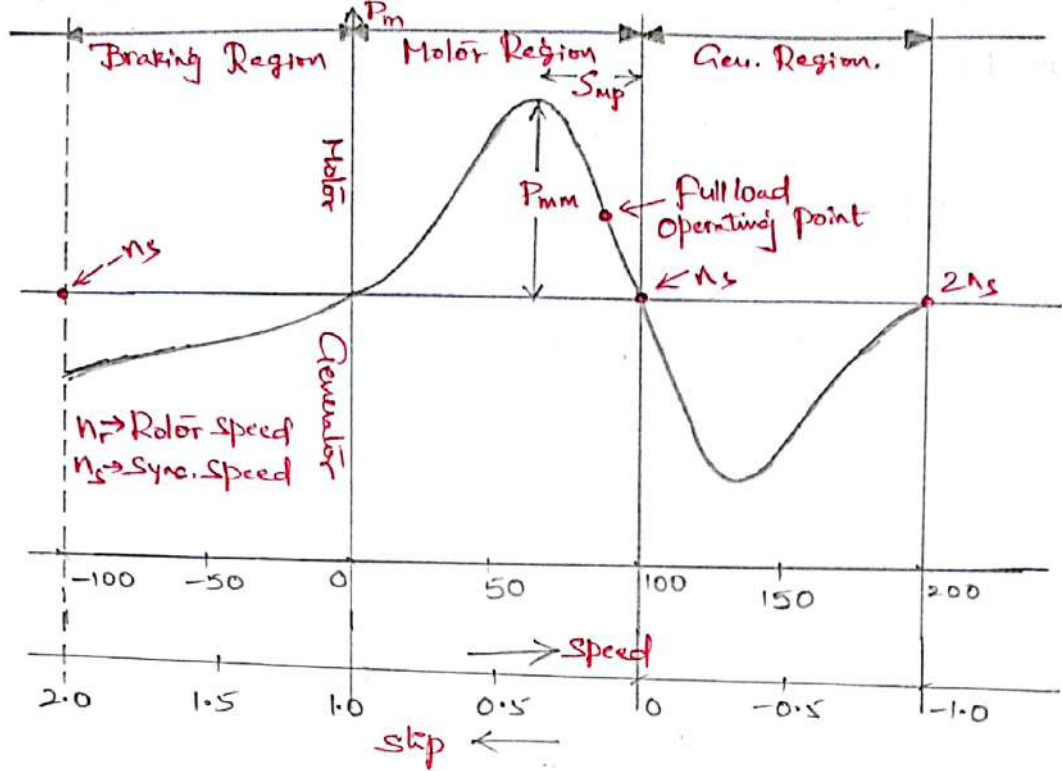
$$\frac{r_2(1-s_{mp})}{s_{mp}} = \sqrt{(R_e + r_2)^2 + x^2}$$

$$(or) \quad s_{mp} = \frac{r_2}{\sqrt{(R_e + r_2)^2 + x^2} + r_2}$$

$$\therefore P_{mm} = \frac{m V_e^2 \sqrt{(R_e + r_2)^2 + x^2}}{2 \left[R_e + r_2 + \sqrt{(R_e + r_2)^2 + x^2} + x^2 \right]}$$

$$P_{mm} = \frac{m V_e^2}{2 \left[R_e + r_2 + \sqrt{(R_e + r_2)^2 + x^2} \right]}$$

It is observed that P_{mm} depends on rotor resistance r_2 whereas T_{em} does not.



Prob. A 3- ϕ star connected, 400V, 50Hz, 4-pole Induction motor has the following per phase constants in ohms referred to stator:

$$r_1 = 0.15, x_1 = 0.45, r_2 = 0.12, x_2 = 0.45, X_m = 28.5$$

Fixed losses = (Core, friction & windage losses) = 400 watt

Compute the stator current, rotor speed, output torque and efficiency when the motor is operated at rated voltage & frequency at a slip of 4%.

Sol.

$$Z_{ab} = \frac{(r_1 + jx_1)(jX_m)}{r_1 + j(x_1 + X_m)}$$

$$= \frac{(0.15 + j0.45)(j28.5)}{0.15 + j(0.45 + 28.5)}$$

$$= (0.145 + j0.444) \Omega$$

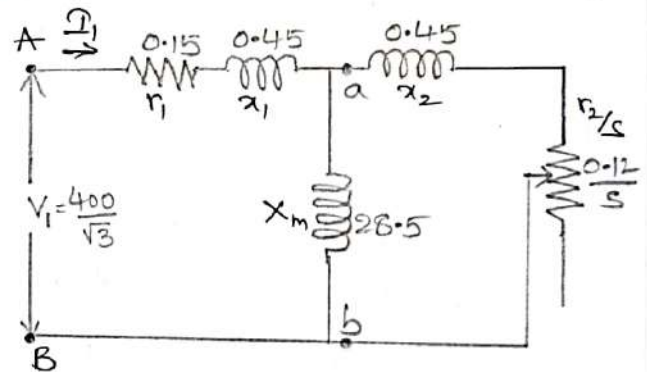
$$Z_{AB} = \frac{(r_2/s + jx_2)(jX_m)}{r_2/s + j(x_2 + X_m)} + r_1 + jx_1$$

$$= \frac{(\frac{0.12}{0.04} + j0.45)(j28.5)}{\frac{0.12}{0.04} + j(0.45 + 28.5)} + 0.15 + j0.45 = 3.25 \angle 21.48^\circ \Omega$$

(i) Stator current, $I_1 = \frac{V_1}{Z_{AB}} = \frac{400/\sqrt{3}}{3.25 \angle 21.48^\circ} = \boxed{71.06 \angle -21.48^\circ \text{ A} = I_1}$

(ii) Synchronous Speed, $N_s = \frac{120f}{P} = \frac{120 \times 50}{4} = 1500 \text{ rpm.}$

Rotor Speed, $N_r = N_s(1 - s) = 1500(1 - 0.04) = \boxed{1440 \text{ rpm.} = N_r}$



(iii) Mech. power developed

$$P_m = (1-s)P_g$$

$$\text{Stator input, } P_1 = \sqrt{3} \times 400 \times 71.06 \times \cos(21.47^\circ) \\ = 45.81 \text{ kW}$$

$$\text{stator losses} = 3 I_1^2 R_1 = 3 \times 71.06^2 \times 0.15 = 2.27 \text{ kW}$$

$$\text{Rotor input, } P_g = P_1 - \text{stator losses} \\ = 45.81 \text{ kW} - 2.27 \text{ kW} \\ = 43.54 \text{ kW}$$

$$P_m = (1-0.04) \times 43.54 \text{ kW} = 41.79 \text{ kW}$$

$$P_{out} = P_m - \text{fixed losses} = 41.79 \text{ kW} - 400 \\ = 41.39 \text{ kW}$$

$$\text{Output torque, } T_{sh} = \frac{P_{out}}{\omega_r} = \frac{41.39 \text{ kW}}{\frac{2\pi \times 1440}{60}} = 274.62 \text{ N-m}$$

$$T_{sh} = 274.62 \text{ N-m}$$

(iv) efficiency, $\eta = \frac{P_{out}}{P_1} \times 100$

$$\% \eta = \frac{41.39 \text{ kW}}{45.81 \text{ kW}} \times 100 \Rightarrow \% \eta = 90.35\%$$

Prob: For the above problem, determine (a) the slip at which maximum torque occurs, the maximum torque and the corresponding output power; (b) the rotor current & Torque at starting; (c) the external resistance to be inserted in the rotor circuit to produce maximum torque at starting; (d) The internal power developed for a slip of 0.04 and (e) the maximum internal power developed and corresponding slip. stator to rotor effective turns ratio is 1.2.

Sol: (a) Slip at Maximum Torque, $S_{MT} = \frac{r_2}{\sqrt{R_e^2 + (x_2 + x_e)^2}}$

$$Z_e = Z_{ab} = (0.145 + j0.444) \Omega = R_e + jx_e$$

$$\therefore S_{MT} = \frac{0.12}{\sqrt{0.145^2 + (0.45 + 0.444)^2}} = 0.1325$$

$$T_{em} = T_{gm} = \frac{K_t}{2[R_e + \sqrt{R_e^2 + x^2}]} ; K_t = \frac{3}{2\pi N_s} \cdot V_e^2$$

$$V_c = \frac{V_1}{\sqrt{3}} = \frac{V_1}{\sqrt{3}} \cdot \frac{jX_m}{j(X_m + x_1)} = \frac{400}{\sqrt{3}} \times \frac{28.5}{28.95} = 227.35 \text{ V}$$

$$K_t = \frac{3}{2\pi \times \frac{1500}{60}} \times 227.35^2 = 987.67$$

$$T_{cm} = \frac{K_t}{2(\sqrt{R_e} + \sqrt{R_e^2 + X^2})}$$

$$R_e = 0.145$$

$$X = x_2 + x_c = 0.45 + 0.444 = 0.894$$

$$T_{cm} = \frac{987.67}{2(\sqrt{0.145} + \sqrt{0.145^2 + 0.894^2})} = 481.77 \text{ N-m.}$$

Corresponding Mech. power developed =

$$P_{sh, max} = T_{max} \times (1 - S_{MT}) \times \omega_s = 481.77 \times (1 - 0.1325) \times 2\pi \times \frac{1500}{60} = 65.62 \text{ kW.}$$

Shaft power, $P_{out} = P_{sh} - \text{Mech. losses}$

$$P_{sh} = 65.62 \text{ kW} - 400 = 65.22 \text{ kW.}$$

$$\text{Maximum Shaft Torque, } T_{sh, max} = \frac{P_{sh}}{\omega_r} = \frac{65.22 \text{ kW}}{2\pi \times \frac{1500}{60} \times (1 - 0.1325)}$$

$$T_{sh, max} = 478.86 \text{ N-m.}$$

Prob: A 3- ϕ , 400/200V, Y-Y Connected wound rotor induction motor has 0.062 rotor resistance and 0.3 Ω stand still reactance per phase. Find the additional resistance required in the rotor circuit to make the starting torque equal to the maximum torque of the motor.

Sol: b.k.T $\frac{T_{st}}{T_{g, max}} = \frac{2S_{MT}}{S_{MT}^2 + 1}$

Given data $\frac{T_{st}}{T_{g, max}} = 1 = \frac{2S_{MT}}{S_{MT}^2 + 1}$; $S_{MT} = \frac{r_2 + r_{ext}}{x_2} = \frac{R}{x_2}$

$$1 = \frac{2 \cdot R/0.3}{(R/0.3)^2 + 1} \Rightarrow (R/0.3)^2 + 1 = \frac{2R}{0.3}$$

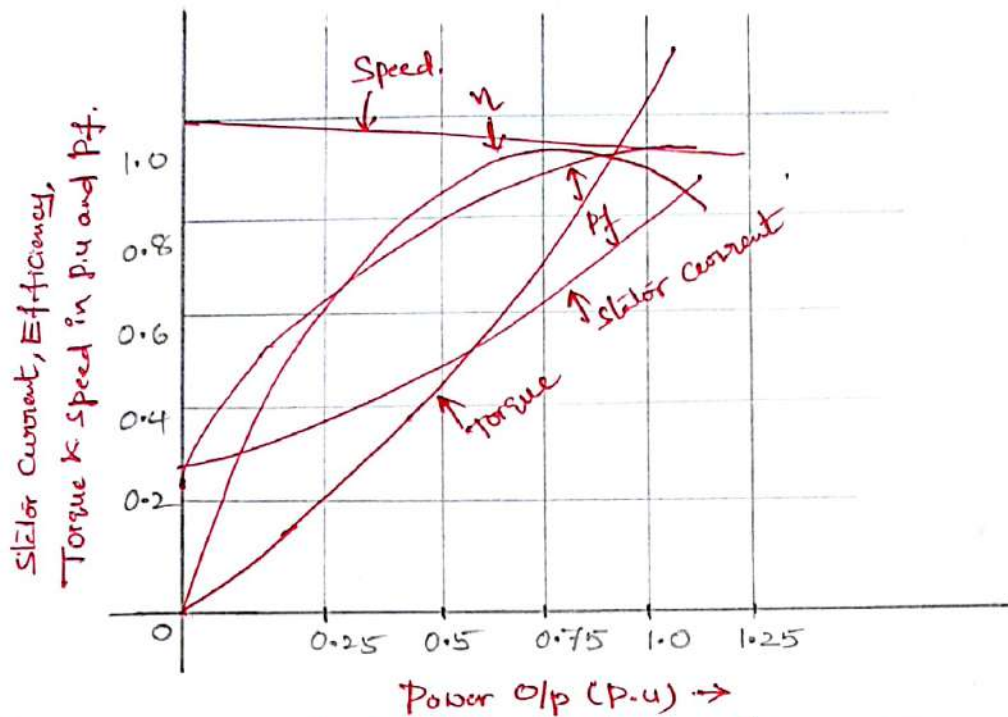
$$11.11R^2 - 6.67R + 1 = 0$$

$$R = 0.3 \Omega = r_2 + r_{ext}$$

$$r_{ext} = R - r_2 = 0.3 - 0.06 = 0.24 \Omega$$

Prob: 3- ϕ , 50Hz, 8-pole IM has full load slip of 2%. The rotor resistance & stand still reactance per phase are 0.01 Ω and 0.05 Ω respectively. Find the ratio of maximum to full load slip torque and speed at which the maximum torque occurs.

* Operating (or Performance) Characteristics of Induction Motors:



* Determination of Equivalent Circuit Parameters:

The Equivalent Circuit parameters of polyphase induction motors can be determined from no-load test, blocked-rotor test and stator winding D.C. resistance.

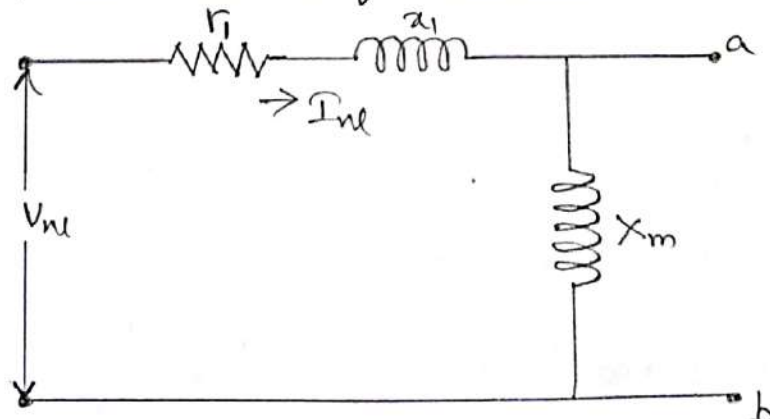
(a) No load test (or Running Light Test):

The induction motor is made to run at no load at rated voltage and frequency. Per Phase values of applied stator voltage V_{nl} , input current I_{nl} and input power P_{nl} are recorded.

The no-load slip s_{nl} is very small, therefore, r_2/s_{nl} is very large as compared to X_m . In view of this, the resultant of parallel branches jX_m and $(\frac{r_2}{s_{nl}} + jx_2)$ is almost equal to jX_m . Thus the no-load reactance X_{nl} seen from the stator terminals is equal to $x_1 + X_m$, i.e.,

$$X_{nl} = x_1 + X_m = X_1$$

Where X_1 is the stator self-reactance



From the instrument Readings at no-load, stator no-load Impedance

$$Z_{nl} = \frac{V_{nl}}{I_{nl}}$$

and stator no-load Resistance, $R_{nl} = \frac{P_{nl}}{I_{nl}^2}$

$$\therefore X_{nl} = \sqrt{Z_{nl}^2 - R_{nl}^2}$$

The Rotational losses P_R (friction, windage loss and Core loss) are usually assumed constant and can be obtained from the relation

$$P_R = m(P_{nl} - I_{nl}^2 r_1)$$

Where m = no. of stator phases

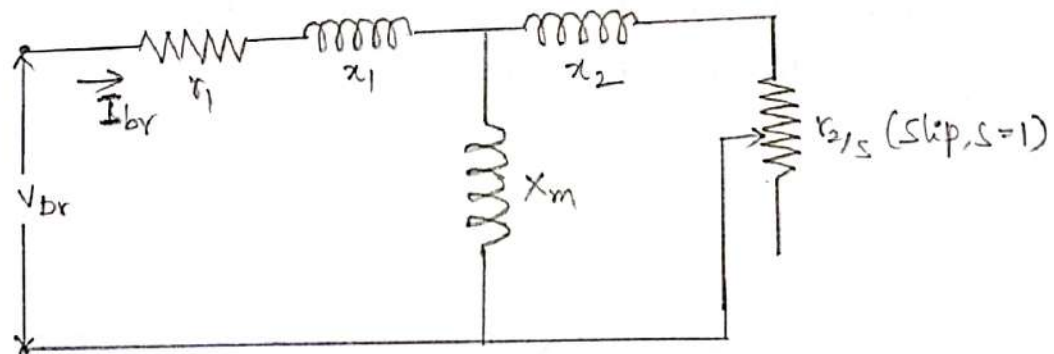
r_1 = Per phase stator Resistance

Thus the no-load test gives X_{nl} and the Rotational losses P_R .

(b) Blocked Rotor Test:

Blocked rotor test, similar to the short circuit test on a transformer, is performed on the induction motor to calculate its leakage impedance.

For performing this test, the rotor shaft is blocked by external means, i.e., the rotor shaft is held stationary by belt-pulley arrangement/by hand. Now balanced poly phase voltage at rated frequency are applied to the stator terminals through a poly phase variac. This applied voltage is adjusted till rated current flows in stator winding. Per phase values of applied voltage V_{br} , input current (= rated current) I_{br} and the input power P_{br} are recorded.



From the instruments readings during blocked-rotor test, the parameters can be obtained as under:

The blocked-Rotor impedance, $Z_{br} = \frac{V_{br}}{I_{br}}$

and blocked Rotor Resistance, $R_{br} = \frac{P_{br}}{I_{br}^2}$, $X_{br} = \sqrt{Z_{br}^2 - R_{br}^2}$

From the above equivalent circuit.

$$\begin{aligned}
 Z_{br} &= R_{br} + jX_{br} = r_1 + jx_1 + \frac{jX_m(r_2 + jx_2)}{r_2 + j(x_2 + X_m)} \\
 &= r_1 + jx_1 + \frac{jX_m(r_2 + jx_2)}{r_2 + j(x_2 + X_m)} \cdot \frac{r_2 - j(x_2 + X_m)}{r_2 - j(x_2 + X_m)} \\
 &= r_1 + jx_1 + \frac{jX_m [r_2^2 - j r_2 x_2 - j r_2 X_m + j r_2 x_2 - j^2 x_2^2 - j^2 X_m^2]}{r_2^2 - j^2 (x_2 + X_m)^2} \\
 &= r_1 + jx_1 + \frac{jX_m [r_2^2 - j r_2 X_m + x_2^2 + x_2 X_m]}{r_2^2 + (x_2 + X_m)^2} \\
 &= r_1 + jx_1 + \frac{jX_m [r_2^2 - j r_2 X_m + x_2 (x_2 + X_m)]}{r_2^2 + (x_2 + X_m)^2} \\
 &= r_1 + jx_1 + \frac{jX_m [r_2^2 - j r_2 X_m + x_2 X_2]}{r_2^2 + X_2^2}
 \end{aligned}$$

where $X_2 = x_2 + X_m$, the rotor self Reactance

$$\therefore X_{br} = x_1 + \frac{X_m [r_2^2 + x_2 X_m]}{r_2^2 + X_2^2} = x_1 + \frac{X_m \left[\frac{r_2^2}{X_2} + x_2 \right]}{\frac{r_2^2}{X_2} + X_2}$$

Since $X_2 \gg r_2$, $\frac{r_2^2}{X_2}$ may be neglected

$$\therefore X_{br} = x_1 + \frac{X_m x_2}{X_2} = x_1 + \frac{X_m x_2}{x_2 + X_m} = x_1 + \frac{x_2}{1 + \frac{X_m}{x_2}}$$

Usually magnetizing Reactance $X_m \gg$ Rotor leakage Reactance x_2
then $\frac{x_2}{X_m}$ is neglected

$$\therefore X_{br} = x_1 + x_2$$

There is no practical method of separation of x_1 & x_2 . For Wound-Rotor induction motors, x_1 is assumed equal to x_2 ; i.e.,

$$x_1 = x_2 = \frac{1}{2} X_{br}$$

Once x_1 is known, the stator magnetizing Reactance is given by

$$X_m = X_{br} - x_1$$

$$R_{br} = r_1 + r_2 \frac{X_m^2}{r_2^2 + X_2^2}$$

$X_2 \gg r_2$, r_2 in the denominator can be neglected compared to X_2

$$R_{br} = r_1 + r_2 \left(\frac{X_m}{X_2} \right)^2 \text{ \& Perphase Rotor Resistance } r_2 = (R_{br} - r_1) \left[\frac{x_2}{X_m} \right]^2$$

Prob: A 10kW, 400V, 4-pole delta-connected Squirrel Cage Induction Motor gave the following test results:

No-load Test: 400V, 8A, 250W

Blocked Rotor Test: 90V, 35A, 1350W.

The D.C resistance of the stator winding per phase measured immediately after the blocked-rotor test is 0.6Ω . Calculate the rotational losses and the equivalent circuit parameters.

Sol: No load Rotational losses are

$$P_r = 250 - 3 \times \left(\frac{8}{\sqrt{3}}\right)^2 \times 0.6 \times 1.2 = 203.92 \text{ W}$$

$$\text{From no-load Test, } Z_{nl} = \frac{400}{(8/\sqrt{3})} = 86.6 \Omega$$

$$R_{nl} = \frac{250}{3 \left(\frac{8}{\sqrt{3}}\right)^2} = 3.91 \Omega$$

$$X_{nl} = \sqrt{Z_{nl}^2 - R_{nl}^2} = 86.51 \Omega$$

$$\text{From blocked Rotor Test, } Z_{br} = \frac{90}{(35/\sqrt{3})} = 4.45 \Omega$$

$$R_{br} = \frac{1350}{3 \left(\frac{35}{\sqrt{3}}\right)^2} = 6.61 \Omega$$

$$X_{br} = \sqrt{Z_{br}^2 - R_{br}^2} = 2.98 \Omega$$

$$\therefore x_1 = x_2 = \frac{X_{br}}{2} = 2.16 \Omega$$

$$X_m = X_{nl} - x_1 = 86.51 - 2.16 = 84.35 \Omega$$

$$X_2 = X_m + x_2 = 86.51 + 2.16 = 88.67 \Omega$$

$$r_2 = (R_{br} - r_1) \left[\frac{X_2}{X_m} \right]^2 = (6.61 - 0.6 \times 1.2) \left[\frac{88.67}{84.35} \right]^2$$
$$r_2 = 0.42 \Omega$$

$$\therefore r_1 = 0.72 \Omega$$

$$x_1 = 2.16 \Omega$$

$$X_m = 84.35 \Omega$$

$$r_2 = 0.42 \Omega$$

$$x_2 = 2.16 \Omega$$

Circle Diagrams

Prob

A 15kW, 400V, 4-pole, 50Hz, 3-phase Star-connected induction motor gave the following test results:

	No-load test	Blocked Rotor Test
Line Voltage	400V	200V
Line current	9A	50A
power input	1310W	7100W

Stator and Rotor ohmic losses at standstill are assumed equal. Draw the induction motor circle diagram and calculate:

- line current, power factor, slip, torque and efficiency at full load.
- Maximum possible power factor and the corresponding line current
- Maximum power output and Maximum power input.
- Slip at which Maximum torque occurs, maximum torque and
- starting torque.

Sol:

No load Test: $V_0 = 400V$

$$I_0 = 9A$$

$$W_0 = 1310W$$

$$\cos \phi_0 = \frac{W_0}{\sqrt{3} V_0 I_0} = 0.21$$

$$\phi_0 = \cos^{-1}(0.21) = 77.87^\circ$$

Blocked Rotor Test: $V_{sc} = 200V$

$$I_{sc} = 50A$$

$$W_{sc} = 7100W$$

$$\cos \phi_{sc} = \frac{W_{sc}}{\sqrt{3} V_{sc} I_{sc}} = 0.41$$

$$\phi_{sc} = \cos^{-1}(0.41) = 65.8^\circ$$

No-load short circuit current, $I_{SN} = \left(\frac{V_0}{V_{sc}}\right) I_{sc}$
Short ckt current with normal voltage $= \left(\frac{400}{200}\right) \times 50 = 100A$

Current Scale, $1cm = 5A$

$$I_0 = 9A = 1.8cm$$

$$I_{sc} = 50A = 10cm$$

$$I_{SN} = 100A = 20cm$$

Now Draw the I_0 with angle ϕ_0 & I_{SN} with angle ϕ_{SN} with respect to Voltage axis. Add a & A, i.e., output line. From the output line draw the power circle with the radius of ac.

No load short circuit power, $W_{SN} = \sqrt{3} V_0 I_{SN} \cos \phi_{sc}$
short ckt power at Normal voltage $= \sqrt{3} \times 400 \times 100 \times 0.41 = 28.40kW$

From the graph $W_{SN} = 8.15cm = 28.40kW = AB$
 $1cm = 3485.25W$

Power scale, $1cm = 3485.25W$

$$\text{Output} = 15 \text{ kW} = 4.3 \text{ cm}$$

Extend the line AB upto AE.

Draw the line from E to P.

(a) Now input current (or) line current, $I_1 = OP = 6.6 \text{ cm} = 33 \text{ A}$.

$$\text{line power factor, } \cos \phi_1 = \cos(30.5) = 0.862 \text{ lag}$$

From the circle diagram

$$\text{Output} = PT = 4.6 \text{ cm} = 16 \text{ kW}, \text{ Full load slip} = \frac{\text{Rotor cu losses}}{\text{Rotor input}} = \frac{0.45}{5.1} = 0.08$$

From the given data stator & rotor losses are equal. The torque line divides stator & rotor cu losses.

From the circle diagram

$$\text{stator cu losses} = SR = 0.2 \text{ cm} = 697 \text{ W}$$

$$\text{Rotor cu losses} = TS = 0.45 \text{ cm} = 1568.4 \text{ W}$$

$$\text{Fixed losses} = RQ = 0.45 \text{ cm} = 1568.4 \text{ W}$$

$$\text{input power, } P_1 = PQ = 5.7 \text{ cm} = 19865.9 \text{ W} = 19.87 \text{ kW}.$$

$$\text{Efficiency, } \eta = \frac{\text{O/P}}{\text{i/p}} \times 100 = \frac{4.6}{5.7} \times 100 = 80.7\%$$

$$\text{Full load Torque, } T_m = \frac{P_2}{\omega_s} = \frac{\text{Rotor i/p}}{\omega_s}$$

$$T_m = \frac{P_2}{\omega_s} = \frac{5.1 \times 3485.25}{2\pi \times 1500/60}$$

$$T_m = 113.21 \text{ N-m}$$

(b) Maximum possible power factor is input power factor, that is 0.862 lag and the corresponding current is $I_1 = 33 \text{ A}$.

(c) For maximum power output, draw a tangent parallel to aA and draw a parallel line to PQ at that tangent point.

$$XX' = 6.6 \text{ cm} = 6.6 \times 3485.25 = 23 \text{ kW}$$

For maximum power input, draw a tangent parallel to aC and draw a parallel line to PQ from that tangent point.

$$ZZ' = 10.1 \text{ cm} = 10.1 \times 3485.25 = 35.2 \text{ kW}$$

(d) For Maximum torque, draw a tangent parallel to torque line and draw a parallel line to PQ from that tangent point.

$$YY' = 8.1 \text{ cm} =$$

$$T_{\text{max}} = \frac{P_2 \text{ at Max. Torque}}{\omega_s} = \frac{8.1 \times 3485.25}{2\pi \times 1500/60} = 179.8 \text{ N-m}$$

$$\text{Slip at Maximum Torque, } S_{\text{MT}} = \frac{\text{Rotor cu loss at Max. Torque}}{\text{Rotor i/p at Max. Torque}}$$

$$= \frac{1.8}{8.1} = 0.22$$

$$(e) \text{ Starting torque, } T_{\text{st}} = \frac{4 \times 3485.25}{2\pi \times 1500/60} = 88.79 \text{ N-m}.$$

Voltage

OP → line current

LOP → input power factor

PO → input power

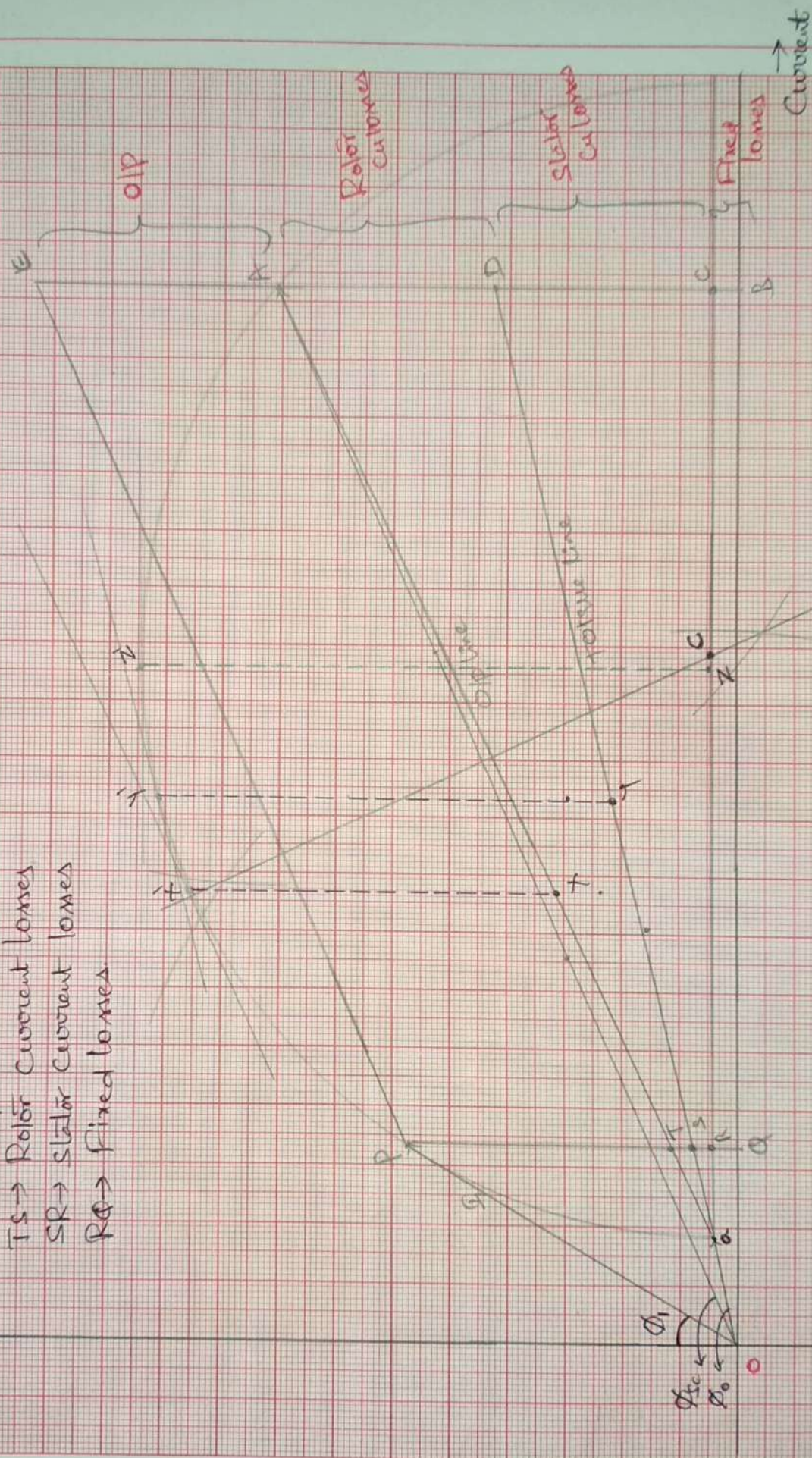
PT → output power

TS → Rotor current losses

SR → stator current losses

RO → Fixed losses

XX' → Maximum O/P power
YY' → Maximum Torque in sync watt



* Power factor Control of 3- ϕ Induction Motors:

The best possible powerfactor of an induction motor operating near its full-load capacity is about 0.86 lag, whereas its no-load powerfactor varies from 0.1 to 0.3 lag. Since induction motors may be light-loaded for a part of their operating time, their average operating power factor is very low. The reasons that an induction motor always operates at lagging power factors for any value of the load, are as follows:

- (i) The magnetizing current, necessary for the production of working magnetic flux, lags the applied voltage by 90° .
- (ii) The voltage drops in stator and rotor leakage reactances increase the power factor angle b/w applied voltage & stator current. Consequently the effect of leakage reactance is to decrease the operating power factor.

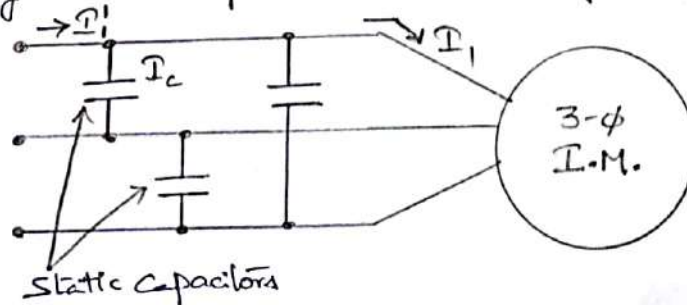
At present, major industrial and agriculture loads consist of polyphase induction motors operating at average low power factors. In order to utilize the distribution and feeder circuits better and to decrease the losses in these circuits, a larger operating power factor is essential.

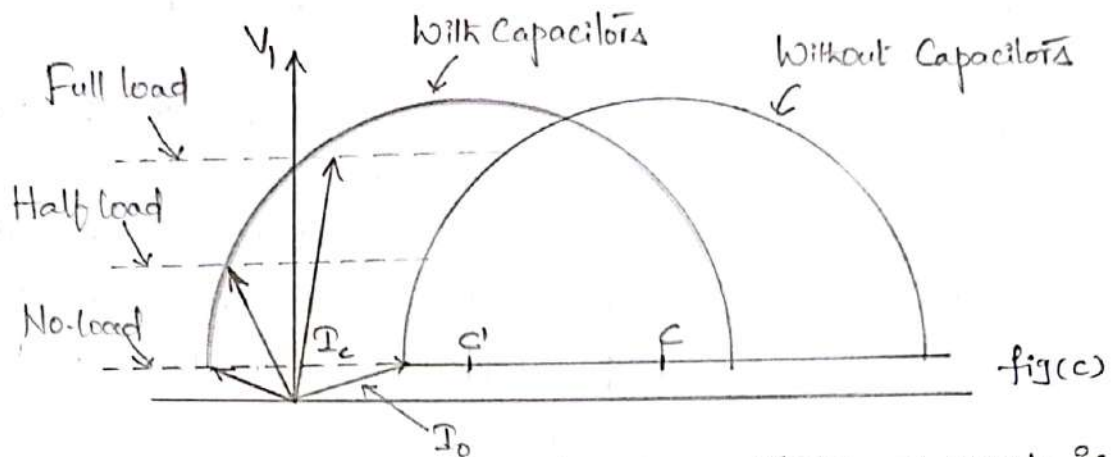
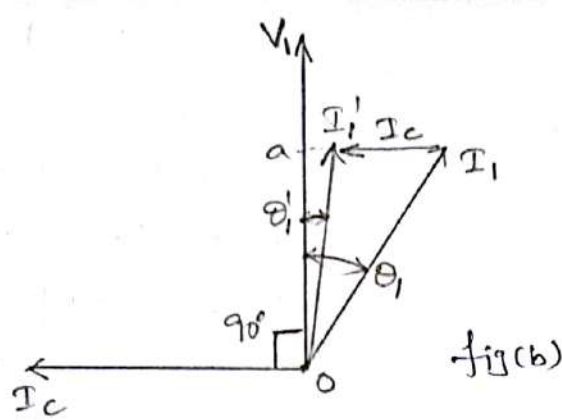
The important methods of powerfactor control of polyphase - induction motors are:

- (a) by the use of static capacitors across the stator terminals of both the squirrel cage and wound rotor induction motors and
- (b) by the use of auxiliary machines for wound-rotor induction motors.

Static Capacitors:

This method, which involves the connection of static capacitors across stator terminals of an induction motor, does not alter the inherent motor characteristics. In smaller sizes, controlling the power factor by static capacitors is the simplest and most economical.





Without the use of Static Capacitors, the stator current is I_1 and the motor operating power factor is $\cos \theta_1$. When Capacitors are connected across the stator terminals shown in fig (c), the current I_c through the capacitors leads the voltage V_1 by 90° .

The degree of power factor correction at different loads can be appreciated with the help of circle diagram.

The leading power factor under certain condition may result in rise of the stator terminal voltage, because the transformer - supplying this induction motor may have negative voltage regulation at these loads.

Sometimes the switching on of induction motor and capacitor as a unit may result in self-excitation which may cause serious stator over voltages or excessive transient torques. The self-excitation of the induction machine can be avoided by limiting the size of the parallel capacitors. A still better way of avoiding the self excitation is to adjust the capacitance with load either manually or automatically.

* Starting of PolyPhase Induction Motors:

From Torque-slip characteristics a 3-phase induction motor has positive finite starting torque. When slip, $S=1.0$. This means that 3- ϕ induction motor is a self-starting motor and begins to rotate on its own when connected to a 3- ϕ source.

At the instant of starting, a 3-phase induction motor behaves like a transformer with short circuited secondary. Consequently, a 3- ϕ induction motor takes high starting current if started at full voltage. In order to limit this high starting current to reasonable values, some methods are adopted for the starting of 3- ϕ induction motors.

Starting of Squirrel-Cage motors:

For cage motors, the choice of any particular method of starting depends on

- (i) Size and design of the motor
- (ii) Capacity of the power lines
- (iii) Type of the driven load.

There are primarily two methods for starting Squirrel cage IMs:

- (a) Full Voltage starting
- (b) Reduced Voltage starting.

The full voltage starting consists of DOL starting only. The reduced voltage starting has the advantage of reducing the starting current, but it produces an objectionable reduction in the starting torque, on account of the fact that motor torque is proportional to the square of the applied voltage.

The reduced voltage starting methods are:

- (i) Stator Resistor (or Reactor) starting.
- (ii) Auto transformer starting
- (iii) Star-Delta starting.

(1) Direct-on-line (across-the-line) starting:

This method involves the direct switching of poly phase stator on to the supply mains. The motor takes low power factor starting current of 5 to 7 times its full-load current, depending upon its size.

and design. Such large currents of short duration don't harm the rugged squirrel cage motor, but the high currents may cause objectionable voltage drop in the power supply lines feeding the induction motor. These large voltage drops cause undesirable dip in the supply voltage; consequently the operation of other equipments connected to the same supply line is affected considerably.

If the supply system is of sufficient power capacity & the low-power factor starting-current surges don't cause objectionable voltage dips in the supply line voltage, then the direct-on-line starting should be preferred.

The relation between starting torque T_{st} and full-load torque T_{fl} is now obtained. Let I_{st} and I_{fl} be the per-phase stator currents drawn from the supply mains corresponding to starting and full-load conditions respectively.

$$\text{Torque } T_g = T_e = \frac{1}{\omega_s} I_{2fl}^2 \frac{r_2}{s} \quad [\text{full load Torque per-phase}]$$

$$\therefore \frac{T_{st}}{T_e} = \frac{I_{2st}^2 \frac{r_2}{1}}{I_{2fl}^2 \frac{r_2}{s}} = \left[\frac{I_{2st}}{I_{2fl}} \right]^2 \cdot s \quad \text{--- (1)}$$

Eqn (1) valid in case rotor resistance remains constant.

If no-load current is neglected, then

$$I_{2st} \times \text{effective stator turns} = I_{2st} \times \text{effective Rotor turns.}$$

$$\frac{I_{st}}{I_{2st}} = \text{effective Rotor to stator turns.}$$

$$\therefore \frac{I_{st}}{I_{fl}} = \frac{\text{effective Rotor to stator turns} \times I_{2st}}{\text{effective Rotor to stator turns} \times I_{2fl}} = \frac{I_{2st}}{I_{2fl}}$$

$$\therefore \text{From eqn (1)} \quad \boxed{\frac{T_{st}}{T_e} = \left(\frac{I_{st}}{I_{fl}} \right)^2 \cdot s} \quad \text{--- (2)}$$

If V_1 is the per phase stator voltage and Z_{sc} is the standstill per phase leakage impedance referred to stator, then per phase short circuit current at standstill is,

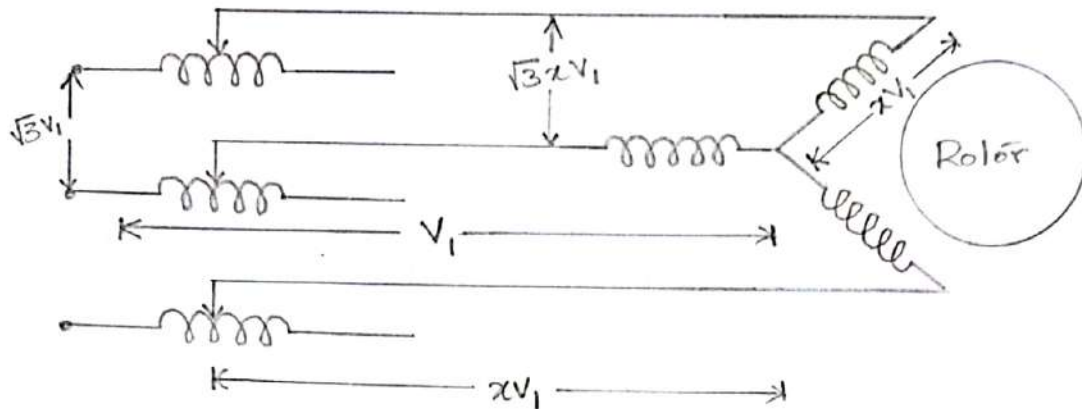
$$I_{sc} = V_1 / Z_{sc} ; Z_{sc} = (r_1 + r_2) + j(\omega_1 + \omega_2)$$

∴ For direct Switching, $I_{st} = I_{sc} = \frac{V_1}{Z_{sc}}$

∴ eq (2) $\Rightarrow \boxed{\frac{T_{st}}{T_e} = \left(\frac{I_{st}}{I_{fl}} \right)^2 \cdot S} \quad \text{--- (3)}$

(b) Stator Resistance (or Reactor) Starting:

In this method, a resistor or a reactor is inserted in between motor terminals and Supply mains.



At the time of starting, some voltage drop occurs across the starting resistor or reactor and, therefore, only a fraction α (less than 1) of the supply voltage appears across the stator terminals. This reduces the per-phase starting current I_{st} drawn by the motor from the supply mains. As the motor speeds up, the reactor is cut out in steps and finally short circuited when the motor speed is near to its operating speed. Since per phase voltage is reduced to αV_1 , the per phase starting current I_{st} is given by

$$I_{st} = \frac{\alpha V_1}{Z_{sc}} = \alpha I_{sc}$$

From eq (3) $\Rightarrow \frac{T_{st}}{T_e} = \left(\frac{I_{st}}{I_{fl}} \right)^2 \cdot S$

$$\boxed{\frac{T_{st}}{T_e} = \alpha^2 \left(\frac{I_{sc}}{I_{fl}} \right)^2 \cdot S}$$

In an induction motor, Torque $\propto (\text{Voltage})^2$

$$\therefore \frac{\text{Starting torque with reactor starting}}{\text{Starting torque with direct starting}} = \frac{(\alpha V_1)^2}{V_1^2} = \alpha^2$$

Series reactor more costly than the series resistor, but the former has lower energy loss and is more effective in reducing the voltage, because the induction motor power factor at starting is quite low.

(C) Auto-transformer starting:

A fraction αV_1 of the Supply Voltage V_1 is applied to the stator terminals at the time of starting, by means of an Auto-transformer. This reduces the motor current and also the current drawn from the supply. After the motor has accelerated near to its operating speed, auto-transformer is disconnected and full line voltage is applied to the induction motor by connecting it directly across the supply mains. Note that here α is less than one.

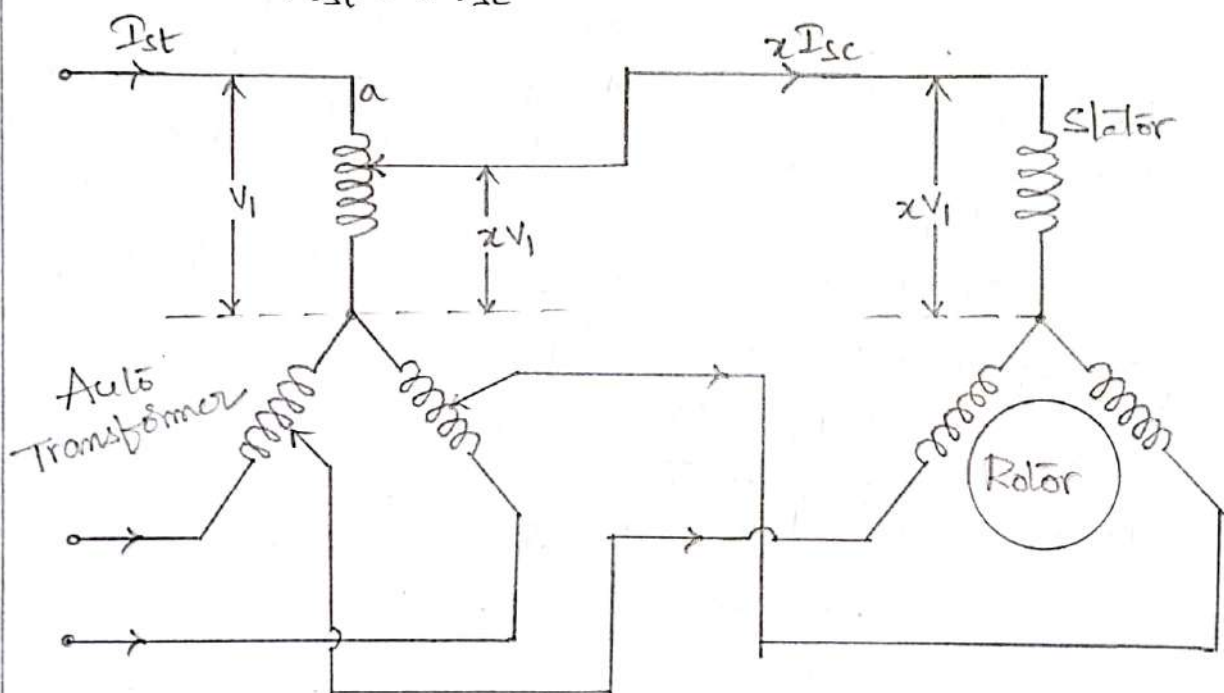
With auto-transformer, per phase starting current in motor winding

$$= \frac{\alpha V_1}{Z_{sc}} = \alpha I_{sc}$$

If no-load current of auto transformer is neglected, then per phase output VA of transformer must be equal to its per phase input VA. That is

$$I_{st} \cdot V_1 = \alpha V_1 (\text{per phase starting current in motor winding})$$
$$= \alpha \cdot V_1 (\alpha I_{sc})$$

$$\therefore I_{st} = \alpha^2 I_{sc}$$



$$\frac{T_{est}}{T_c} = \left(\frac{\alpha I_{sc}}{I_{fl}} \right)^2 \cdot S = \left(\alpha \frac{I_{sc}}{I_{fl}} \right)^2 \cdot S$$
$$= \alpha^2 \left(\frac{I_{sc}}{I_{fl}} \right)^2 \cdot S$$
$$= \alpha^2 \left[\frac{1}{\alpha^2} \frac{I_{st}}{I_{fl}} \right]^2 \cdot S = \frac{1}{\alpha^2} \left(\frac{I_{st}}{I_{fl}} \right)^2 \cdot S$$

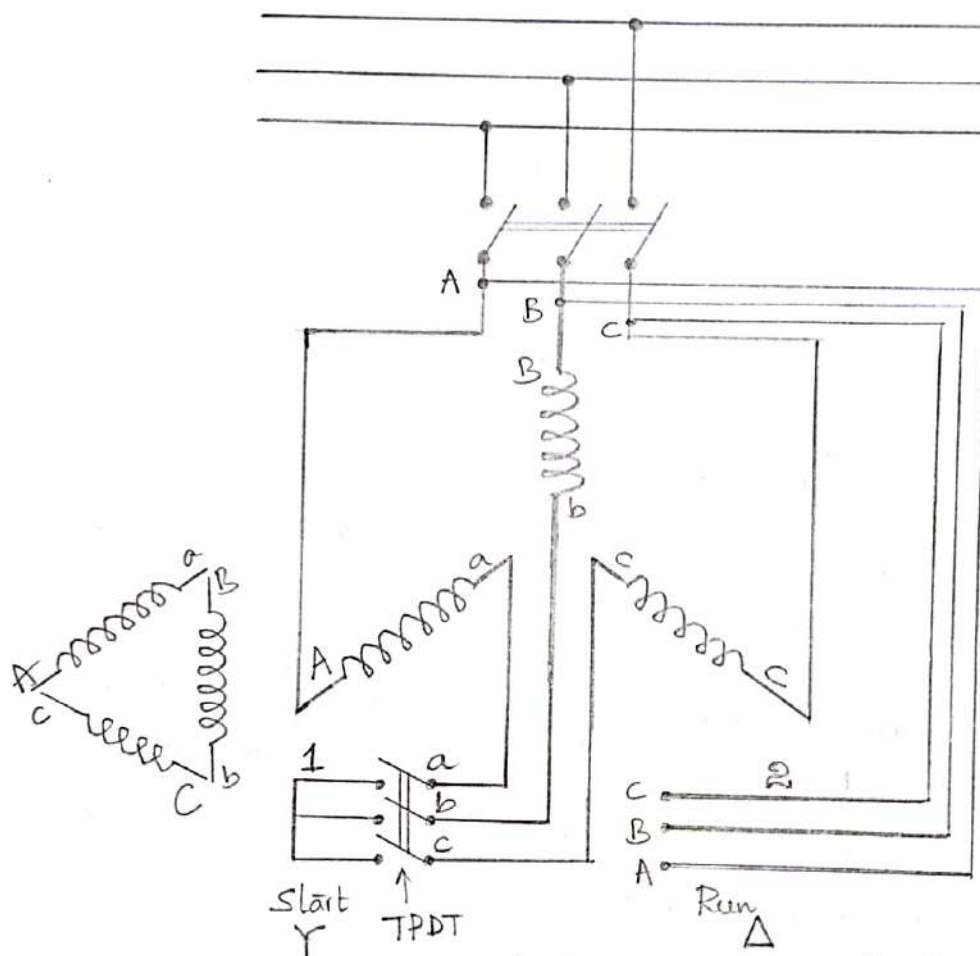
$$\frac{T_{st}}{T_e} = \frac{1}{\alpha^2} \left(\frac{I_{st}}{I_{fl}} \right)^2 \cdot S$$

For an induction motor, torque $\propto (\text{Voltage})^2$

$$\frac{\text{Test with an auto transformer}}{\text{Test with direct switching}} = \left(\frac{2V_1}{V_1} \right)^2 = \alpha^2$$

(d) Star-Delta Starting:

This method of starting is used for motors designed to operate normally in delta. The six terminals from the three phases of the stator must be available as shown in figure.



The stator phases are first connected in star by throwing the triple pole double throw (TPDT) switch to position 1 and after steady-state speed has reached, TPDT is thrown over to position 2, thereby connecting the stator winding in delta.

In position 1, terminals a, b, c are short circuited. In position 2, terminal a is connected to B; b to C, c to A.

Let V_L be the line voltage. The per phase motor starting current $I_{st,Y}$ with stator winding in star is given by $I_{st,Y} = \frac{V_L}{\sqrt{3} Z_{sc}}$

With stator winding in star, $I_{st,y}$ is also the starting line current with star-Delta starter. If the stator winding were delta-connected, then with direct switching, the per phase motor starting current $I_{st,d}$ would be given by

$$I_{st,d} = V_L / Z_{sc} = I_{sc,d}$$

and starting line current with direct switching would be equal to $\sqrt{3} V_L / Z_{sc} = \sqrt{3} I_{st,d} = \sqrt{3} I_{sc,d}$.

$$\begin{aligned} \frac{\text{Starting line current with star-delta starter}}{\text{Starting line current with direct switching in delta}} &= \frac{I_{st,y}}{\sqrt{3} I_{st,d}} \\ &= \frac{\frac{1}{\sqrt{3}} \frac{V_L}{Z_{sc}}}{\sqrt{3} \cdot \frac{V_L}{Z_{sc}}} = \frac{1}{3}. \end{aligned}$$

Thus with star-delta starter, the starting current from the mains is one-third of that with direct switching in delta. Also,

$$\frac{\text{Starting torque with star-delta starter}}{\text{Starting torque with direct switching in delta}} = \frac{(V_L / \sqrt{3})^2}{V_L^2} = \frac{1}{3}.$$

This shows that star-Delta starter also reduces the starting torque to one-third of that produced by direct switching in delta.

$$\frac{\text{Starting torque with star-Delta starter, } T_{st}}{\text{Full load torque with stator winding in Delta, } T_{fl,d}} =$$

$$= \frac{\frac{1}{w_s} (I_{st,y})^2 \frac{r_2}{s}}{\frac{1}{w_s} (I_{fl,d})^2 \frac{r_2}{s}} = \frac{(\frac{1}{\sqrt{3}} I_{st,d})^2}{(I_{fl,d})^2} s = \frac{1}{3} \left(\frac{I_{sc,d}}{I_{fl,d}} \right)^2 s.$$

Star-Delta starter is cheap as compared to Auto transformer starter and is, therefore, used extensively. For applications requiring starting torque not exceeding about 50% of rated torque, star-delta starting is the most suitable method. This method is, therefore, used for starting SCIM coupled with pumps, fans, propellers, machine-tools, motor-generator sets etc.

For line voltages exceeding 3.3 kV, star-delta starter is not used, because then the stator winding in delta require large turns making the motor more expensive.

Prob: A 3- ϕ squirrel cage induction motor has a short circuit current of 5 times the full load current. Its full load slip is 5%. Calculate the starting torque as a percentage of full load torque if the motor is started by
(i) DOL starter (ii) Star Delta starter. (April/May-2017, Set-1).

Sol:

Given $\frac{I_{sc}}{I_{fl}} = 5$

slip = 5% = 0.05

(i) Started by DOL starter:

$$I_{st} = I_{sc} = \frac{V_1}{Z_{sc}}$$

$$\frac{T_{st}}{T_e} = \left(\frac{I_{sc}}{I_{fl}} \right)^2 \times S = 1.25$$

$$\therefore T_{st} = 1.25 T_e = 125\% T_e$$

starting torque is 125% of Full load torque.

(ii) Started by Star-Delta starter:

$$\frac{T_{st}}{T_e} = \frac{1}{3} \left(\frac{I_{sc,d}}{I_{fl,d}} \right)^2 \times S = \frac{1}{3} \times 5^2 \times 0.05 = 0.417$$

$$T_{st} = 0.417 T_e$$

Starting torque is 41.7% of Full load torque.

Prob:

The short circuit current of squirrel cage induction motor on normal voltage is 3.5 times the full load current and the full load slip is 4%. Determine the percentage tapping required to an auto transformer starter to start the motor against $\frac{1}{3}$ rd full load torque. Neglect the magnetizing current. (April/May-2017, Set-2)

Sol:

Given $\frac{I_{sc}}{I_{fl}} = 3.5$ (at normal voltage, V_1)

slip = 4%

$$\frac{T_{st}}{T_e} = \frac{1}{3}$$

$$\frac{T_{st} \text{ with an auto-transformer}}{T_{e,fl}} = \frac{1}{x^2} \left(\frac{I_{st}}{I_{fl}} \right)^2 \times S = x^2 \left(\frac{I_{sc}}{I_{fl}} \right)^2 \times S$$

$$\frac{1}{3} = x^2 \times 3.5^2 \times 0.04 \Rightarrow x^2 = \frac{1}{3 \times 3.5^2 \times 0.04} = 0.68$$

$$x = 0.82$$

The required tapping for auto-transformer is 0.82

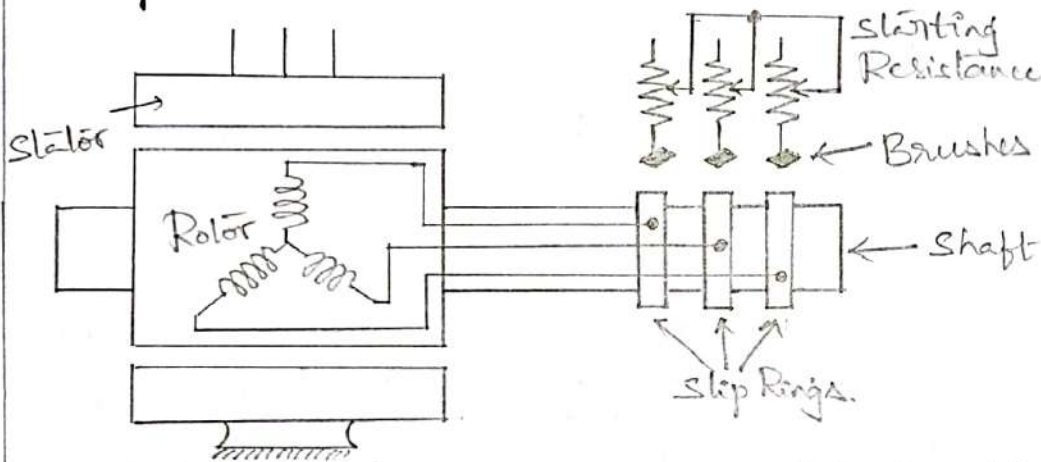
Starting of Slip-Ring Motors:

(1) Rotor Resistance Starting:

The simplest and cheapest method of starting wound-rotor induction motor is by means of added rotor resistance, with full-line voltage across the stator terminals. The addition of external resistance in the rotor circuit of a wound-rotor IM

- (i) decreases its starting current
- (ii) increase its starting torque (for a suitable external resistance)
- (iii) improves its starting power factor.

The Rotor winding terminals of a wound-rotor motor are connected to three slip rings mounted on but insulated from the shaft. The leads, from the three brushes pressing on these slip rings, are taken to external resistances as shown in figure.



At the time of start, the entire resistance is added in the rotor circuit. As the rotor speeds up, the external resistance is decreased in steps so that motor torque tends to remain maximum during the accelerating period. Finally, under normal operation, the external resistance is fully cut off and the slip rings are short circuited so that motor now develops full-load torque at low value of slip for which it is designed.

Prob: An 8-pole, 50Hz, 3- ϕ slip ring induction motor has effective resistance of 0.08Ω /phase. The speed corresponding to maximum torque is 650 rpm. What is the value of resistance to be inserted in rotor circuit to obtain maximum torque at starting.

Sol: Poles, $P = 8$, $f = 50\text{Hz}$

$$N_s = \frac{120f}{P} = \frac{120 \times 50}{8} = 750 \text{ rpm}$$

* Speed Control of an Induction Motor:

The speed of the induction motor is given by

$$N = N_s(1-s) \quad \& \quad N_s = \frac{120f}{P}$$

$$\text{slip, } s = \frac{\text{Rotor cu loss, } P_c}{\text{Rotor input, } P_2}$$

Therefore, the speed of an induction motor can be controlled From the stator side:

1. By changing the supply voltage
2. By changing the supply frequency.
3. By changing the number of stator poles.

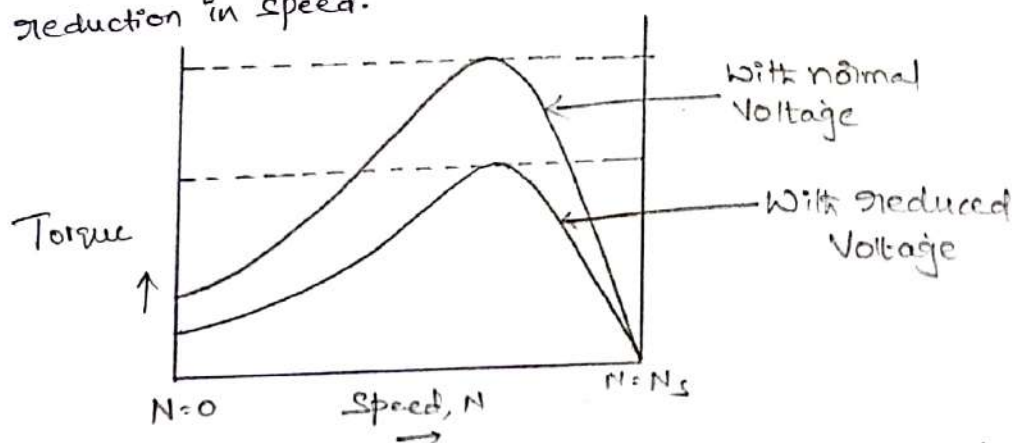
From the rotor side:

4. By changing rotor circuit resistance (slip-ring I.M. only).
5. By injecting voltage in to the rotor circuit
6. By Cascade Control.

1. By changing Supply Voltage:

The torque developed by an induction motor is proportional to square of supply voltage ($T_e \propto E_2^2$)

If supply voltage is reduced below rated value, torque also decreases. To produce the same torque for supplying the same load, slip increases, i.e., motor running at low speed to decrease in supply voltage. The maximum torque developed is reduced considerably with the slight reduction in speed.



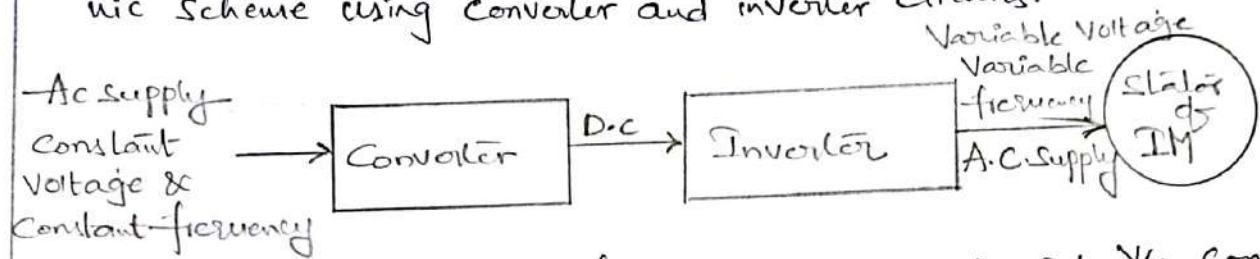
This method is simple and cheapest method but it is rarely used because a large change in voltage is required for small change in speed.

This method is used on small motors driving fans whose torque is proportional to the square of speed.

(2) By Changing Supply Frequency: (V/f Control):

A change of supply frequency changes the synchronous speed and provides a direct method of speed control. To keep the air gap flux constant, the supply voltage must be changed so that ratio of voltage to frequency (V/f) remains constant. Otherwise, the magnetic circuit may become saturated resulting in excessive magnetization current. Therefore, the method is also called as V/f control of the motor.

Hence, in this method, the supply to the induction motor is variable voltage, variable frequency supply and can be achieved by an electronic scheme using converter and inverter circuits.



By selecting the proper frequency and maintaining V/f constant, smooth speed control of the motor is possible.

(3) Changing Number of Stator Poles:

By changing number of poles, synchronous speed is changed and therefore, speed of the motor is changed.

The method is very much suitable for squirrel cage motors only as cage motor automatically adjusts itself to any number of stator poles. In slip ring, the same arrangement is required on the rotor side to match the stator and rotor poles. The disadvantage of this method is step changes in speed and smooth speed control is not possible.

The number of stator poles can be changed by

- (i) Multiple stator windings
- (ii) Consequent pole Technique.

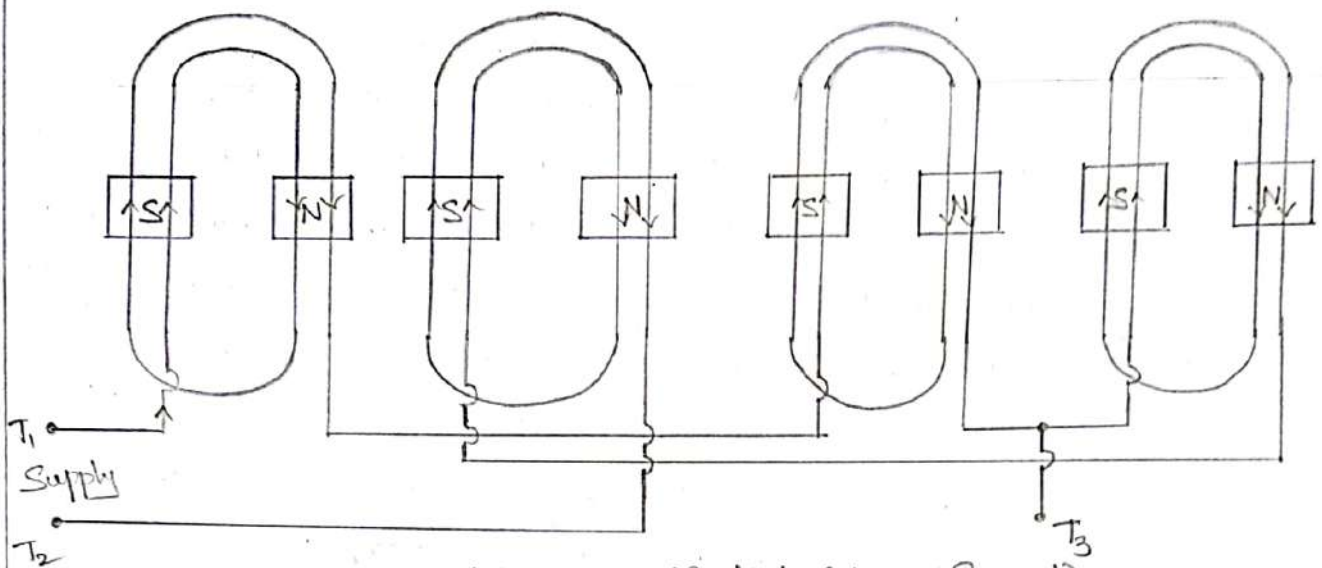
(i) Multiple Stator Windings:

The stator has two separate windings with different number of poles, two definite motor speeds are obtained. These windings are insulated and only one winding is energized at a time.

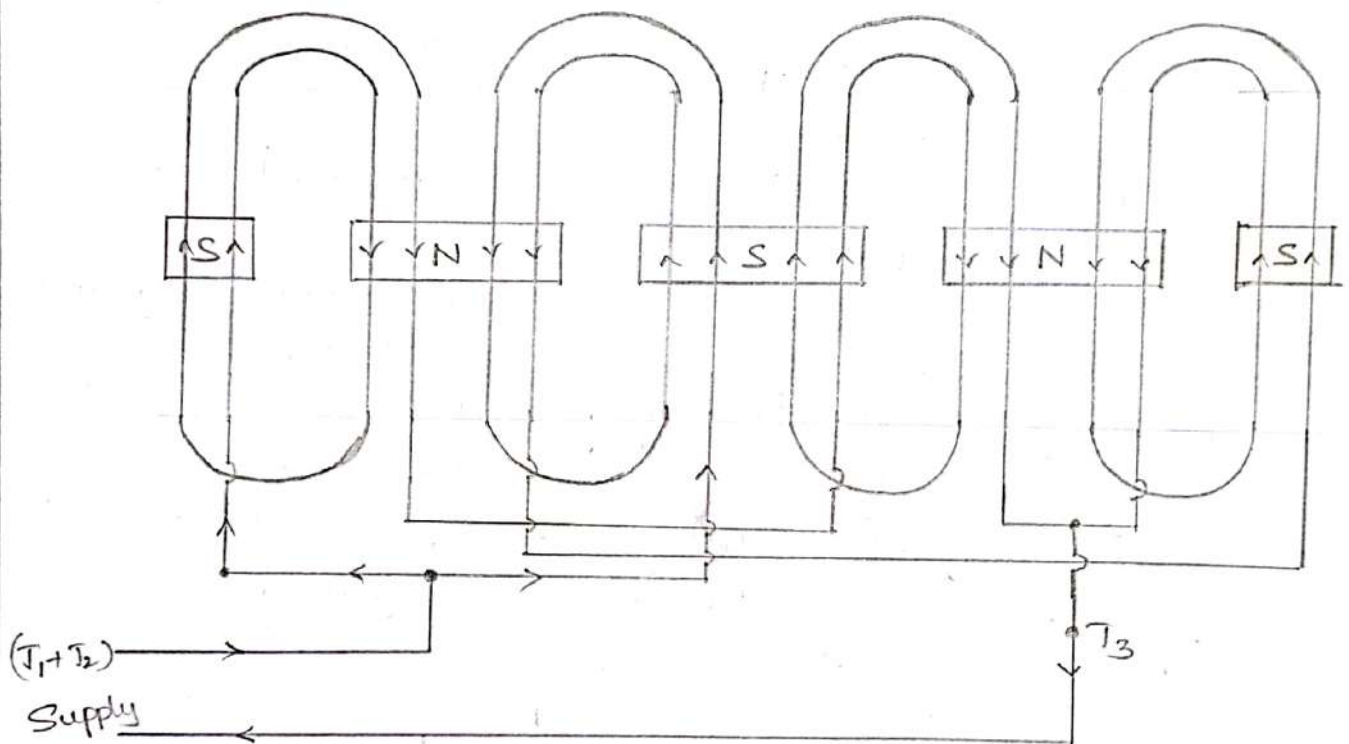
(ii) Consequent pole Technique:

In this method, the number of poles can be changed in the ratio of 2:1 by changing the connections of the stator winding.

Consider the 1- ϕ of a certain winding having 3 lapping points. When the supply is across the two terminals (T_1 & T_2) and the 3rd terminal is left free, the distribution of current will be as shown in figure (a).



(a) 8-pole Winding (Low Speed)

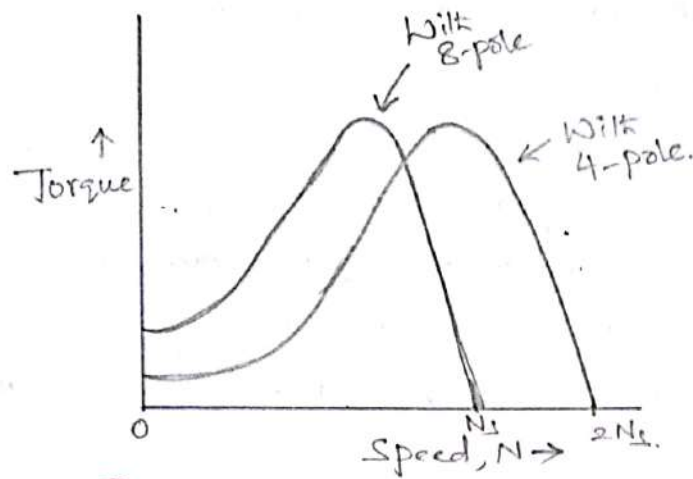


(b) 4-pole Winding (High Speed)

The Connections of fig(a) is of 8-pole Winding.

Now if these two terminals (T_1 & T_2) are joined together and on observing the direction of current, it will be found that now there are 4-poles only as shown in figure (b). Thus the Speed now will be double of the previous value.

With Combination of having two separate stator windings and using consequent pole technique, four different speeds are possible.



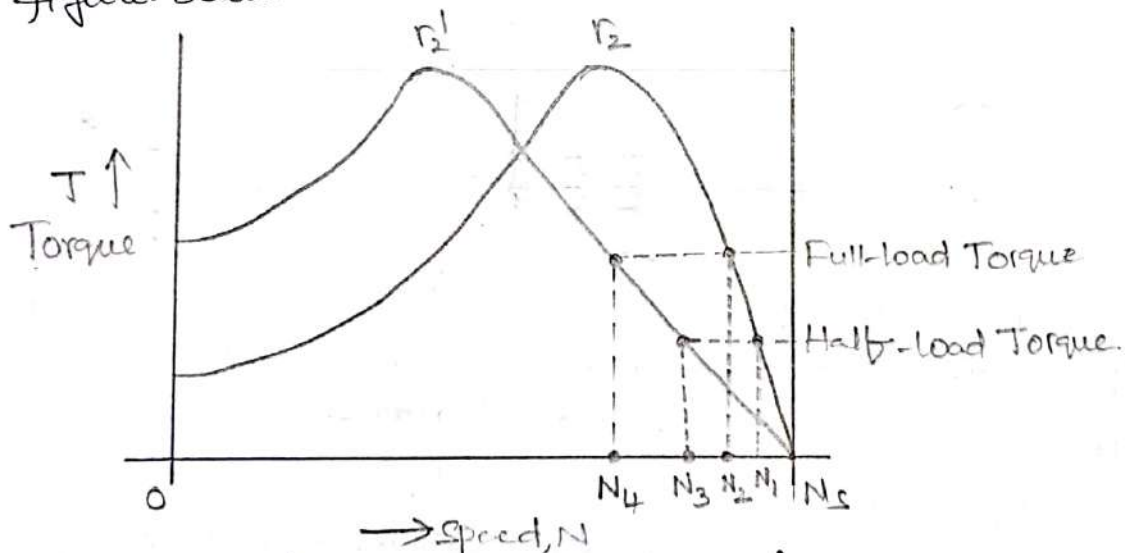
(4) By changing Rotor Circuit Resistance:

This method is applicable only to slip ring Induction motors. In Squirrel Cage motors, rotor circuit resistance can not be varied.

By increasing rotor resistance (inserting external resistance in the motor circuit), the slip increases and hence speed decreases.

$$\text{Since } \text{Slip} = \frac{\text{Rotor cu loss}}{\text{Rotor input}}$$

However, addition of resistance in the rotor circuit increases the rotor cu losses which reduces the efficiency. Therefore, this method should be used only when speed control is required for a short period. Also this method gives poor speed regulation i.e., the variation of the speed with variation of load becomes higher is shown in figure below.



From above figure, Change of Load from Half load to Full load

1. Variation of speed is from N_1 to N_2 with R_2 as rotor Resistance
2. Variation of Speed is from N_3 to N_4 with R_2 as rotor Resistance, which is much higher Compared with R_2 as rotor resistance.

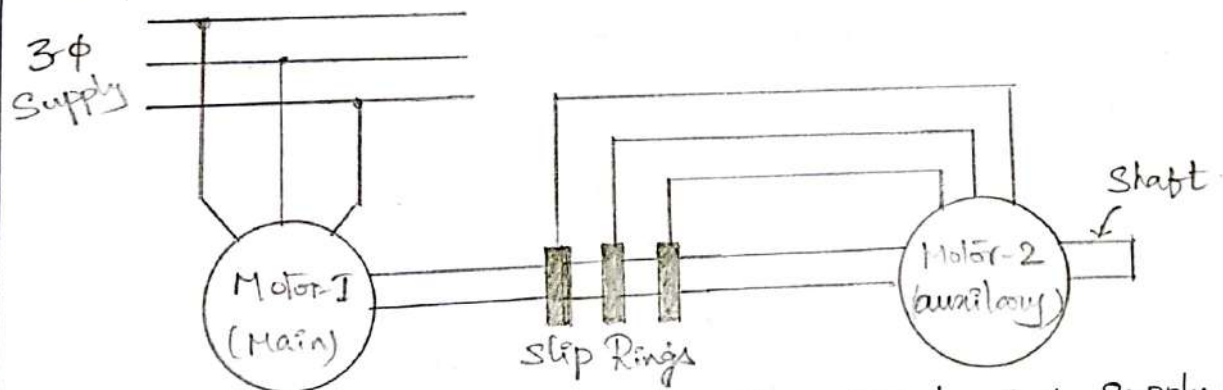
(5) By Injecting Voltage into the Rotor Circuit:

The speed of a IM can be controlled by injecting a voltage of slip frequency into the rotor circuit (i.e., across the slip rings). If the injected voltage is in phase with the rotor induced emf is equivalent to decreasing the rotor resistance, hence slip decreases or speed increases. If injected voltage is in phase opposition to the rotor induced emf is equivalent to increasing the rotor resistance, hence slip increases or speed decreases.

A large and smooth variation of speed is possible by this method. However, it requires an auxiliary machine capable of injecting the correct slip frequency voltage into the rotor circuit. Therefore, it is costlier & used with motors of very large rating.

(6) By Cascade Control:

This is also called as Concatenation or Tandem operation of IM's. In this method, two motors are required, at least one of which must have a slip ring type called as Main Motor while the second motor is called as Auxiliary Motor. The two motors are mechanically coupled as shown in figure.



The stator of main motor is connected to 3- ϕ supply while the auxiliary motor is fed from the rotor circuit of main motor. This is called as "Cascading of the Motors".

If the torque produced by both act in the same direction, Cascading is called "Cumulative Cascading". If torques produced are in opposite direction, Cascading is called as "Differential Cascading".

Let P_1 : No. of poles of main motor
 P_2 : No. of poles of Auxiliary motor
 f : Supply frequency

Due to cascading four different synchronous speeds are possible

(i) Main motor is connected to supply independently

$$\text{So, } N_s = \frac{120f}{P_1}$$

(ii) Auxiliary motor is connected to supply independently with main motor disconnected from the supply

$$\text{So, } N_s = \frac{120f}{P_2}$$

(iii) In Cascade Combination, Cumulative Cascading produces effective number of poles as $P_1 + P_2$

$$\therefore N_s = \frac{120f}{P_1 + P_2}$$

(iv) In Differential Cascading, the effective number of poles are $P_1 - P_2$

$$\therefore N_s = \frac{120f}{P_1 - P_2}$$

$$\therefore N_s = \frac{120f}{P_1 + P_2}$$

Synchronous speed of motor-1 $\Rightarrow N_{s1} = \frac{120f}{P_1}$

Let S_1 and S_2 be the slip of motors 1 & 2.

Now the frequency of motor-2 is $S_1 f$

Synchronous speed of motor-2 $\Rightarrow N_{s2} = \frac{120 S_1 f}{P_2}$

Actual speed of motor-1, $N_1 = N_{s1} (1 - S_1) = \frac{120f}{P_1} (1 - S_1)$

Actual speed of motor-2, $N_2 = N_{s2} (1 - S_2)$

$$= \frac{120 S_1 f}{P_2} (1 - S_2)$$

As the two motors are mechanically coupled, the two speeds must be same. Hence

$$\frac{120f}{P_1} (1 - S_1) = \frac{120 S_1 f}{P_2} (1 - S_2)$$

$$\frac{(1 - S_1)}{P_1} = \frac{(S_1 - S_1 S_2)}{P_2}$$

$$\frac{1 - S_1}{P_1} = \frac{S_1}{P_2} \quad (\because S_1 S_2 \text{ is very small and neglected})$$

$$S_1 P_1 + S_1 P_2 = P_2 \Rightarrow S_1 = \frac{P_2}{P_1 + P_2}$$

Hence, speed of the set,

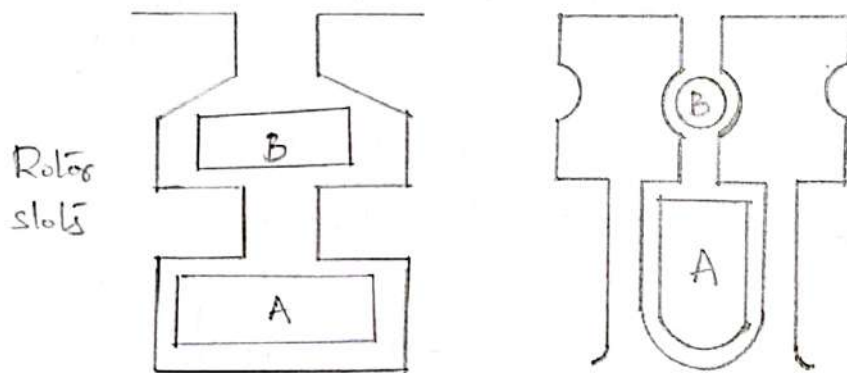
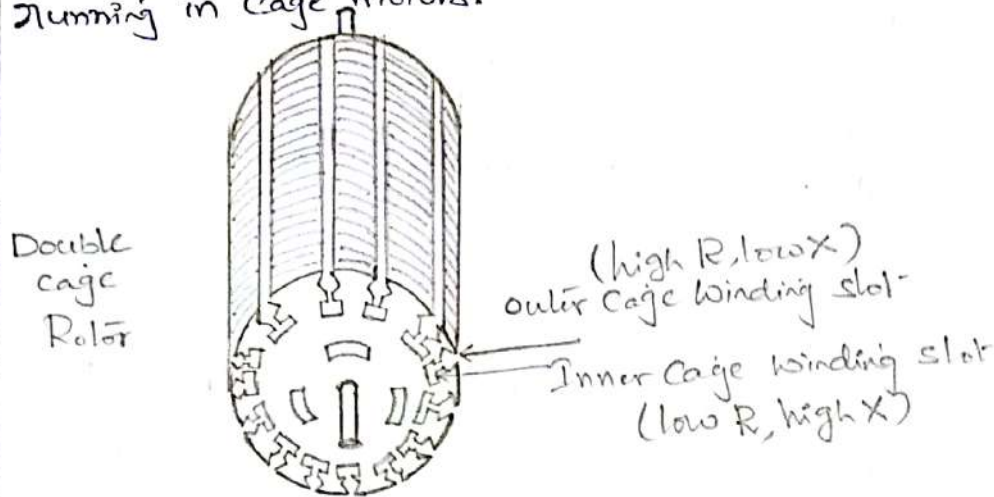
$$N = \frac{120f}{P_1} (1 - S_1) = \frac{120f}{P_1} \left[1 - \frac{P_2}{P_1 + P_2} \right] = \frac{120f}{P_1 + P_2}$$

$$\text{Cascade speed of the set} = \frac{120f}{P_1 + P_2}$$

* Double Cage Induction Motor:

The starting torque in a Squirrel cage motor is small - because of its low rotor resistance and there is no way to insert external resistance as the winding is permanently short circuited.

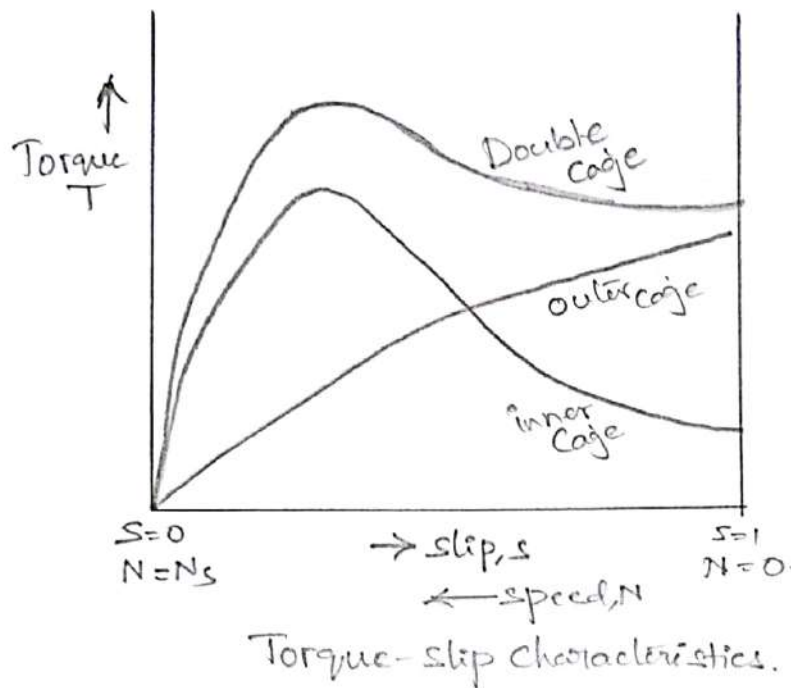
The starting torque of the squirrel cage motors are improved by using a double cage motor i.e., a method of obtaining a comparatively high rotor resistance at starting and low rotor resistance when running in cage motors.



The double cage motor consists of two separate cages on the rotor as shown in figure. The inner cage (made up of bars A) is deeply embedded in the rotor and has bars of large cross section, i.e., low resistance. As it is deeper in the rotor iron it has also a higher inductance (reactance). The outer cage (made up of bars B) is near the surface and has bars of small cross section. Its reactance is low and resistance is high.

At starting, the frequency of the rotor induced emf or current is high. So that the reactance of the inner cage ($= 2\pi fL$) and therefore its impedance are both high. Hence, very little current flows in it. Most of the rotor current will flow through the high resistance outer cage and high starting torque is obtained.

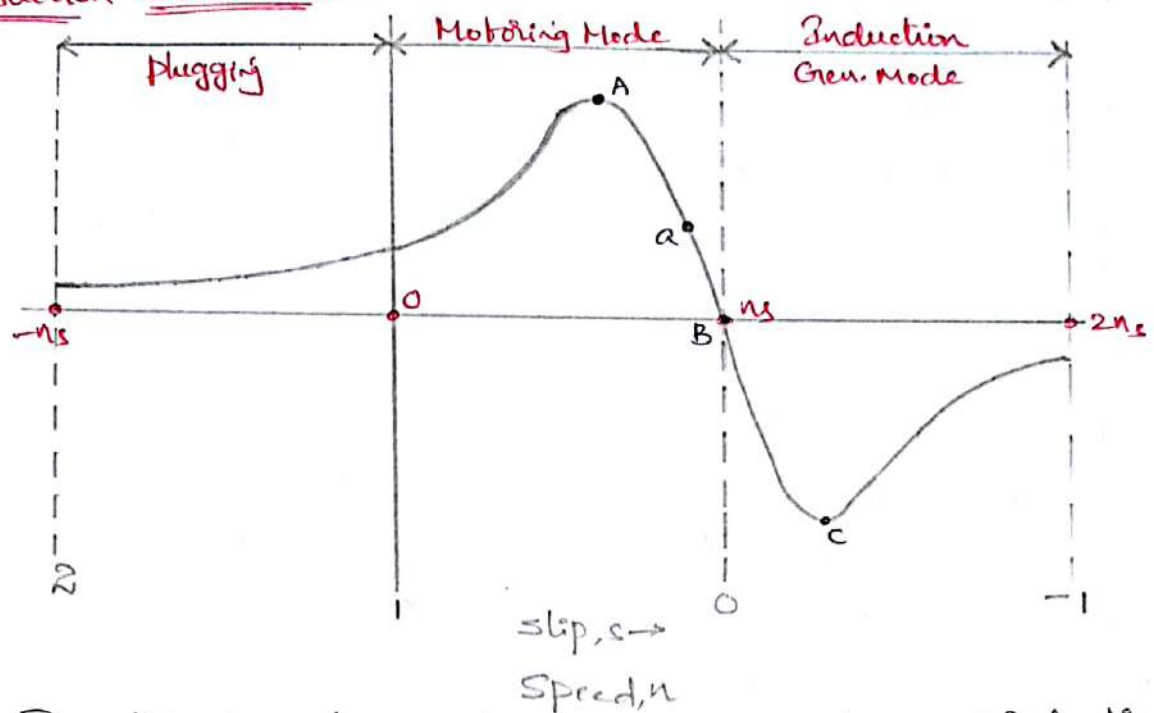
As the speed increases, the rotor frequency decreases. Under normal conditions, the reactance and hence the impedance of inner cage decreases. Most of the rotor current then flows through the inner low resistance cage, thus the I^2R loss in the rotor circuit will not be high and therefore giving a high efficiency under running condition.



Advantages:

1. It permits high starting torque and good efficiency during running conditions
2. Useful for frequent starting under heavy load conditions.
3. No need of maintaining sliprings and brushes when compared to wound rotor.

* Induction Generator:



From the above torque-slip characteristics of a 3- ϕ induction motor over its entire slip range of -1 to 2 . During the motoring mode, the operating point lies somewhere near point 'a' between the points A and B. Here B' indicates zero torque at zero slip and A denotes maximum torque at slip s_{mt} .

Suppose that 3- ϕ induction motor coupled to a prime-mover whose speed can be controlled and that the operating point is 'a'. If the prime mover speed is increased, the operating point begins to travel from 'a' towards 'B'. When prime-mover speed attains the synchronous speed n_s , slip is zero and operating point is 'B'. For prime-mover speed above synchronous speed, slip becomes negative. As a consequence, motor emf and motor current, torque and power all attain negative values. This implies that for negative slip, electric torque developed is negative, i.e. opposite to the prime-mover torque. Under such a condition, machine must act as a generator and delivers its generated emf/power to the supply mains from which it was taking power when working as a 3- ϕ induction motor.

Just as point A denotes the maximum torque of a 3- ϕ IM, point C indicates the maximum torque that can be applied to the induction machine now working as a induction generator. If the prime mover exceeds this maximum torque given by point C, induction generator will overspeed at some higher value of slip, the generating effect vanishes.

Applications of Induction generators:

These are run in parallel with the existing 3- ϕ system. Thus, Voltage & frequency of induction generators cannot be controlled. Only the active power can be regulated through the speed control of prime-mover driving the 3- ϕ induction generator. These generators don't hunt. The short-circuit current of these generators is limited. It is because a short-circuit at once reduces the excitation and thus short-circuit current gets limited.

This induction generators are also used in unattended small remote hydro plants thereby interconnecting a small power station to a large power distribution network.

Their disadvantages are as under:

- (i) They require considerable amount of reactive power from the existing supply system.
- (ii) Efficiency is relatively poor.
- (iii) It can work at leading power factor only.

Induction generator principle is used for regenerative braking of hoists or electric locomotives driven by 3- ϕ Induction motors.

* Applications of Polyphase Induction Motors:

Squirrel cage motors with relatively low rotor resistance are used for fans, centrifugal pumps, most machinery tools, wood-working tools etc. Cage rotor motors with relatively high rotor resistance are used for compressors, crushers, reciprocating pumps. Squirrel cage motors with still higher values of rotor-circuit resistance are used for intermittent loads like punching presses, shears, hoists, elevators etc.

A wound rotor induction motor is used for loads requiring severe starting conditions or for loads requiring speed control. A wound rotor motor is more expensive than the squirrel cage motor and also it requires more maintenance because of the brushes and slip rings. A wound rotor motor also called slip-ring motor, may be used for hoists, cranes, elevators, compressors etc.

The relative advantages of cage motor over a wound-rotor motor of the same power rating, are given below:

- (i) Cage motor requires considerably less conductor material than a wound motor, consequently I^2R loss in cage motor is less. Therefore, cage motor is a little more efficient than a wound-motor motor.
- (ii) Wound motor construction requires slip rings, brushes, short-circuiting devices etc. As a result of it, a wound-motor motor is costlier than a cage induction motor.
- (iii) Cage motor is more rugged and requires no slip rings, brushes etc., therefore, its maintenance charges are low.
- (iv) Cage motor can be cooled better because of its bare end-rings.

The disadvantages of cage motor as compared to a wound motor motor are its small starting torque for very large starting current and its poor starting power factor. In addition to it, the total energy lost during starting of cage motor is much more than with the wound motor motor and this fact is very important where frequent starting of large number of motors are required.

Prob: The motor of a 4-pole, 50 Hz, slip ring IM has a resistance of $0.3 \Omega/\phi$ & runs at 1440 rpm at full load. Calculate the external resistance per phase which must be added to lower the speed to 1320 rpm, the torque being the same as before. [$r = 0.6 \Omega$].

Prob: A certain 3- ϕ , 6 pole, 50 Hz IM when fully loaded, runs with slip of 3%. Find the value of the resistance necessary in series per phase of the motor to reduce the speed by 10%. Assume the resistance of the motor per phase is 0.2Ω . [$r = 0.65 \Omega$].

Prob: Two 50 Hz, 3- ϕ IM having 6 & 4 pole respectively are cumulative cascade, the 6-pole motor being connected to the main supply. Determine the frequencies of the motor currents & the slip referred to each stator field if the set has a slip of 2%.

Prob: A 4-pole & 6-pole IM are connected in cumulative cascade. The frequency in the secondary circuit of a 6-pole motor is observed to be 1 Hz. Determine the slip in each machine & the combined speed of the set. Take supply frequency as 50 Hz.

Prob: A 1100 V, 50 Hz, delta connected IM has a star-connected slip ring motor with a phase transformation ratio of 3.8. The motor resistance and standstill leakage reactance are 0.012Ω & 0.25Ω per phase respectively. Neglecting stator impedance and magnetising current determine:

- (1) The rotor current at start with slip rings shorted.
- (2) The rotor power factor at start with slip rings shorted.
- (3) The rotor current at 4% slip with slip rings shorted.
- (4) The rotor power factor at 4% slip with slip rings shorted.
- (5) The external rotor resistance per phase required to obtain a starting current of 100A in the stator supply lines.

Sol: Transformation Ratio = $1/3.8$

$$\text{Rotor phase voltage at stand still} = 1100 \times \frac{1}{3.8} = 289.5 \text{ V}$$

$$(i) \text{ Rotor impedance / phase} = \sqrt{0.012^2 + 0.25^2} = 0.2503 \Omega$$

$$\text{Rotor phase current at start} = \frac{289.5}{0.2503} = 1157 \text{ A}$$

$$(ii) \text{ Rotor power factor at start, } = \frac{r_2}{Z_2} = \frac{0.012}{0.2503} = 0.048 \text{ lag}$$

$$(iii) \text{ Rotor Impedance at 4\% slip} = \sqrt{0.012^2 + (0.04 \times 0.25)^2} = 0.0156 \Omega$$

$$\text{Rotor current at 4\% slip} = \frac{SE_2}{Z_2} = \frac{0.04 \times 289.5}{0.0156} = 742.3 \text{ A}$$

$$(iv) \text{ Rotor Power factor at 4\% slip} = \frac{0.012}{0.0156} = 0.77 \text{ lag}$$

$$(v) \text{ Required starting current} = 100 \text{ A}$$

$$\text{At the starting current of 100 A in the stator supply line} = I \times 3.8 \\ = 100 \times 3.8 = 380 \text{ A}$$

$$\therefore \text{ Impedance} = \frac{289.5}{380} = 0.7618 \Omega$$

$$\therefore R_2 = \sqrt{Z^2 - X^2} = \sqrt{0.7618^2 - 0.25^2} = 0.7196 \Omega$$

$$R_2 = r_2 + r_{add} \Rightarrow r_{add} = 0.7076 \Omega$$

Prob: The resistance & reactance per phase of the rotor of a 3- ϕ IM are 0.6 Ω and 5 Ω respectively. The IM has a star connected rotor and when the stator is connected to supply of normal voltage, the induced emf between the slip rings at standstill is 80V. Calculate the current in each phase and the power factor at starting when (i) the slip rings are shorted (ii) slip rings are connected to star connected resistance of 4 Ω per phase. [9.17 A, 0.1194 lag, 6.8 A, 0.6765 lag]

Prob: A 12-pole, 3- ϕ , 600V, 50Hz, star-connected, induction motor has rotor-resistance and standstill reactance of 0.03 Ω & 0.5 Ω per phase respectively. Calculate: (a) Speed of maximum torque (b) ratio of full load torque to maximum torque, if the full load speed is 495 rpm.

Single-Phase Motors

Single-phase motors are designed to operate from a 1- ϕ supply, are manufactured in a large number of types to perform a wide variety of useful services in home, offices, factories, workshops and in business establishments etc. They are used only in small sizes, less than 2HP and mostly in the fractional horse power range. Single phase motors are relatively simple in construction. 1- ϕ motors operate at low power factors and are relatively inefficient as compared to three phase motors.

Single-phase motors are classified as:

1. Single-phase Induction motors:

- (a) split phase type
- (b) Capacitor split phase type
- (c) Shaded pole type

Single-phase Induction motors are widely used for fans, washing machines, refrigerators, blowers, centrifugal pumps etc. They are used in very small sizes in toys, hair dryers etc.

2. Single-phase Synchronous motors:

- (a) Reluctance type
- (b) Hysteresis type

These are used in applications such as clocks, recording instruments, teleprinters etc. Where a constant speed is required.

3. Single-phase Commutator motors:

- (a) AC Series motors
- (b) Universal motors

Motors of the series type can be used with either a d.c. supply or a 1- ϕ a.c. supply. These motors provide high starting torque and can operate at high speed. They are widely used in kitchen equipment, portable tools and vacuum cleaners.

4. Special purpose Motors:

- (a) stepper Motors
- (b) Servo Motors.

* Single phase Induction Motor:

Constructionally, this motor is, more or less, similar to polyphase - induction motor, except that

- (i) its stator is provided with a single-phase winding
- (ii) a centrifugal switch is used in some type of motors, in order to cut-out a winding, used only for starting purposes.

It has distributed stator winding and a squirrel cage rotor. When fed from a single-phase supply, its stator winding produces a flux which is only alternating i.e., one which alternates along one space axis only. It is not a synchronously rotating flux, as in the case of a two- or a three phase stator winding fed from a 2 or 3-phase supply. Now an alternating or pulsating flux acting on a stationary squirrel cage rotor cannot produce rotation. This is why a single phase motor is not self starting.

However, if the rotor of a such machine is given an initial start by hand or otherwise, in either direction, then immediately a torque arises and the motor accelerates to its final speed.

This peculiar behaviour of the motor has been explained in two ways: (i) by two field (or) Double field revolving theory.
(ii) by cross field theory.

Double field revolving theory:

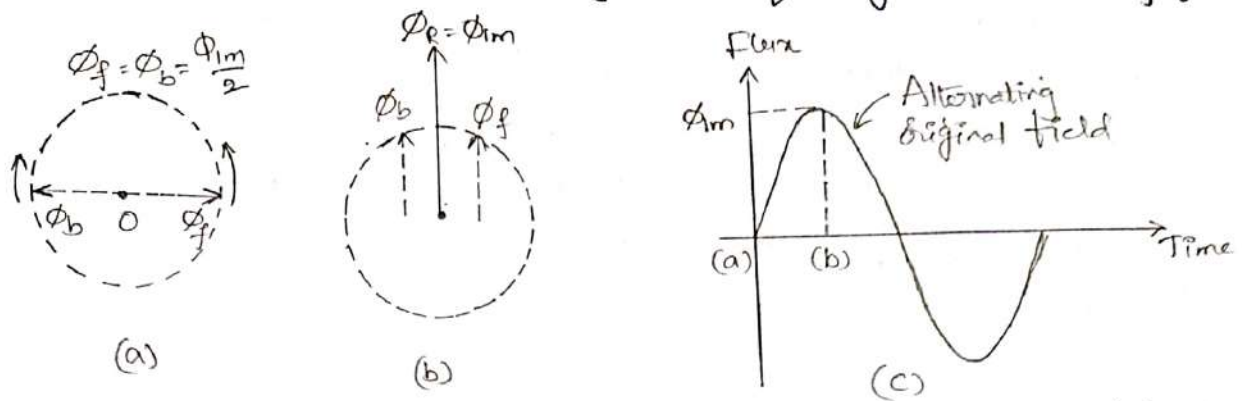
An alternating field produced by the stator winding can be represented by sum of two revolving fields of equal magnitude and are rotating in opposite directions at synchronous speeds. The magnitude of each field is equal to one half of the maximum value of the alternating field, i.e., $\phi_m/2$ where ϕ_m is the maximum value of alternating field.

Let anti clockwise rotating field is ϕ_f say forward field while the clockwise rotating field be ϕ_b say backward field. The resultant of these two fields at any instant gives instantaneous value of the main alternating field at that instant.

Consider the different instances as shown below:

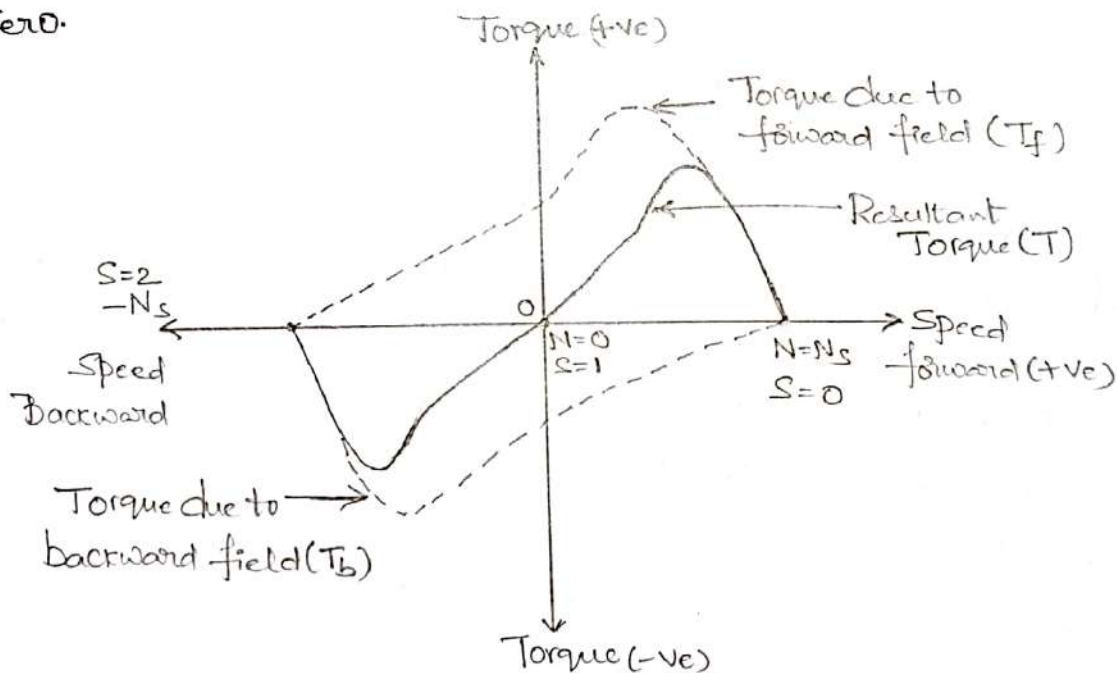
- (i) The two fields are shown opposite to each other at stand in fig (i). The resultant magnitude field, $\phi_R = 0$.
- (ii) The fig (ii) shows that after 90° , the fields are rotated in such a way that both of them are now pointing in the same direction.

The resultant magnetic flux, $\phi_R = \phi_f + \phi_b = \frac{\phi_{im}}{2} + \frac{\phi_{im}}{2} = \phi_{im}$
 This instant gives maximum magnitude of original alternating field.



So the Continuous rotation of these two fields (components) gives the original stator magnetic field which is shown in fig(c). This is purely alternating in nature.

Now each separate component is rotating and produces a torque. So one component ϕ_f produces a torque in anti clockwise direction and second component ϕ_b produces a torque in clockwise direction. If anti clockwise torque is assumed positive then clockwise torque produced by another is negative. The resultant of the two torques at start is zero.



Torque-Speed curve for both components and Resultant

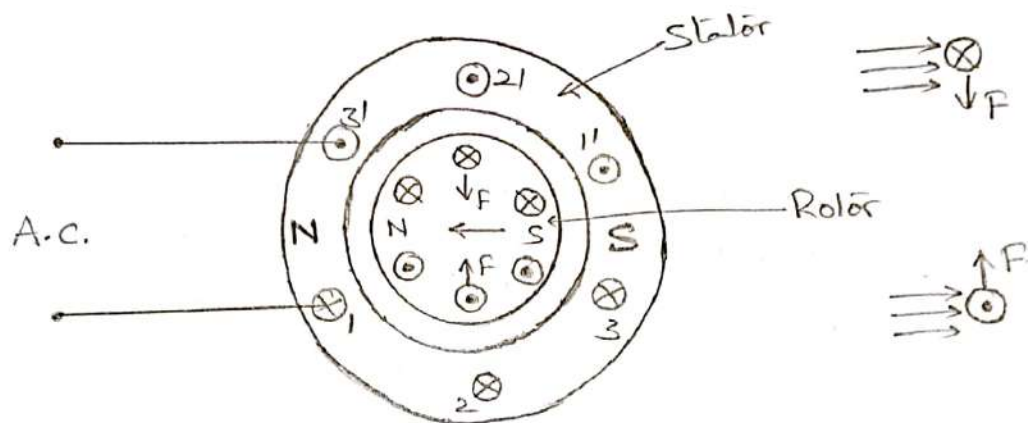
Cross field Theory:

Motor at Standstill:

Consider Standstill Conditions with the stator winding connected to a 1- ϕ AC supply. The stator current establishes a field act along the horizontal axis is shown in figure. The stator field is alternating in polarity and varying sinusoidally with time. The alternating field will

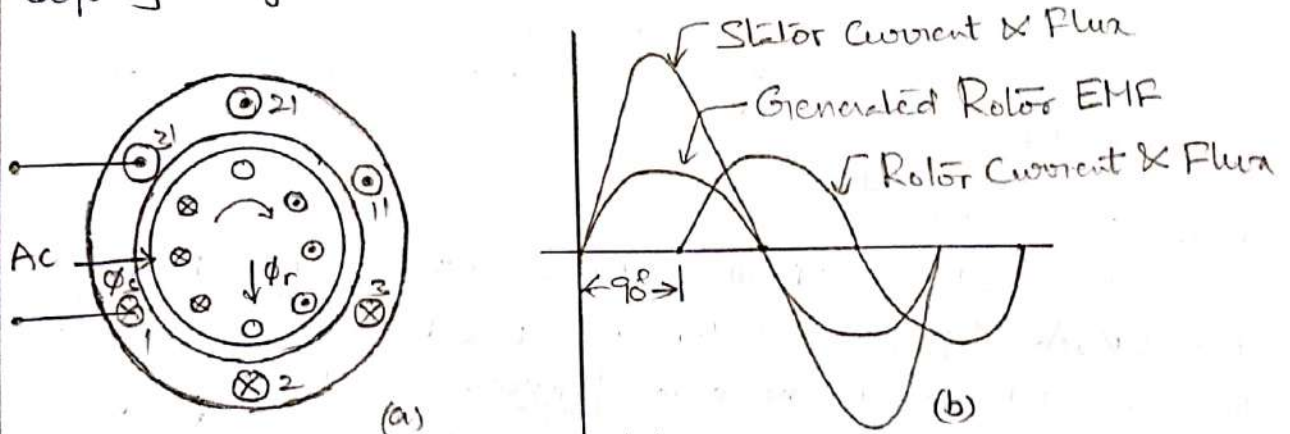
Will induce an emf in the rotor winding by transformer action. This emf will cause a current to flow in the rotor winding.

The directions of currents in the rotor conductors is shown in figure. The rotor current establish poles on the rotor surface and these are in direct line with the stator poles. The axis of the stator and rotor fields are aligned. The forces on the rotor conductors in top half are in downward direction, whereas the forces on the rotor conductors in bottom half are in upward direction. The two sets of forces will cancel and the rotor will experience no torque.



Motor Running:

When however, the rotor is made to rotate say in the clockwise direction by some external means, the rotor conductors cut across the stator field, causing an emf to be generated in them. The direction of the emfs are determined by Fleming's Right hand rule, will be outward in one side of the vertical axis and inward in the other side of the vertical axis as indicated by the dots and crosses as shown in figure (a). The generated rotor emf vary in phase with the stator current and flux. The rotor current due to these emfs lags by nearly 90° owing to low R & high X of the rotor winding.



The field produced by the rotor currents is at right angles to the field by the stator current hence it is known as Cross field. Thus the stator field, Φ_s and rotor field, Φ_r are in space and time Quadrature. These two fields will produce a resultant revolving field which will rotate in the direction in which the rotor was given an initial rotation. Hence torque is exerted on the rotor and the motor continues to rotate.

From the above discussion

- (1) At standstill there can be no cross field only the pulsating stator field and therefore the inherent starting torque of a Φ induced motor is zero.
- (2) If, however, the rotor is made to run by some external means, then it will continue to develop torque in the direction of rotation.

* Types of Single-Phase Induction Motors:

Single phase induction motors are not self starting due to absence of rotating magnetic field. It is not practicable to provide initial rotation to the motor, manually. So in practice some provision is made to provide rotating magnetic field at start, to make the motors self starting.

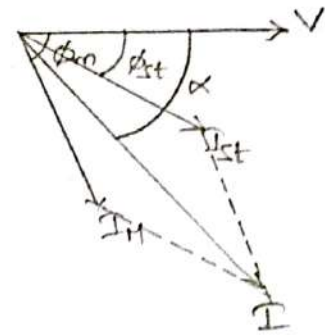
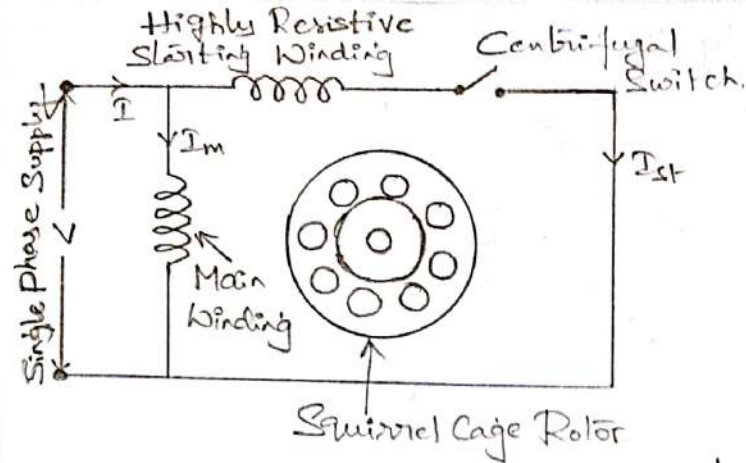
Single phase motors are classified according to the methods of starting used for them.

1. Split phase motor
2. Capacitor start motor
3. Capacitor motor (or) permanent split capacitor motor
4. Capacitor start capacitor run motor (Two value capacitor motor)
5. Shaded pole motor

* Split phase Motor (or) Resistance split phase Motor:

In this motor, in addition to normal stator winding called as Main winding, stator carries one more winding called as auxiliary or starting winding. The two windings are displaced in space by 90° with each other. Mostly the rotor is of squirrel cage rotor.

The time displacement between the two currents is obtained by designing the auxiliary winding as highly resistive in nature.



The main winding is reactive hence carries current, I_m which lags supply voltage by large angle while auxiliary winding carries a current, I_{st} which almost inphase with supply voltage. Hence there exists a phase difference between the two alternating current- as shown in phasor diagram. The motor is equivalent to an unbalanced two-phase motor.

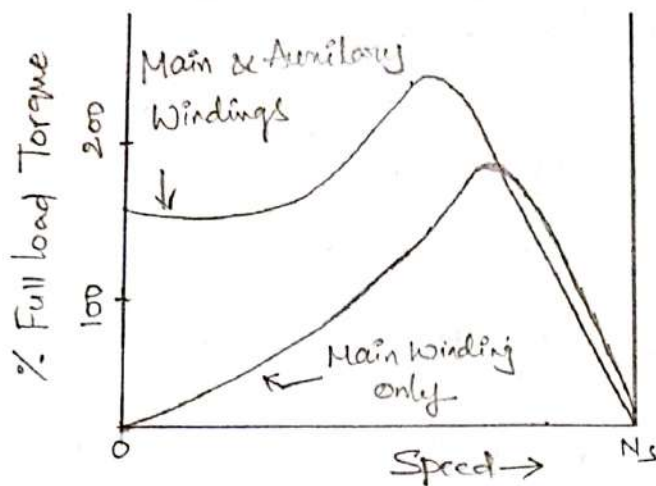
The two currents produce the two fluxes which also have a phase difference of ' α ' as shown in the phasor diagram. Due to these two fluxes having considerable phase difference in between them, a rotating magnetic field is produced and the motor develops a starting torque.

Because of the high resistance character of auxiliary wdg, this motor is also known as Resistance start motor. The auxiliary wdg is not designed for continuous operation to prevent the motor from drawing excessive currents from the line and also to protect the winding from damage due to overheating. Hence the auxiliary winding is automatically disconnected by the centrifugal switch, when the motor picks up the speed (75 to 80% of synchronous speed).

As auxiliary winding is in the circuit only at start, hence it is also called as starting winding. As currents I_m and I_{st} are splitted from each other by angle ' α ' at start, the motor is called as split phase motor. The starting torque is proportional $\sin \alpha$. This motor has moderate starting torque generally it is 125 to 150% of the F.L Torque.

Reversal of Rotation: Motors can be reversed by reversing the terminals of either main winding or auxiliary winding.

Applications: These motors are used in fans, blowers, washing machines, oil burners, electronic tools etc.



Torque - speed curve of split phase motor

* Equivalent Circuit of a Single-phase induction motor

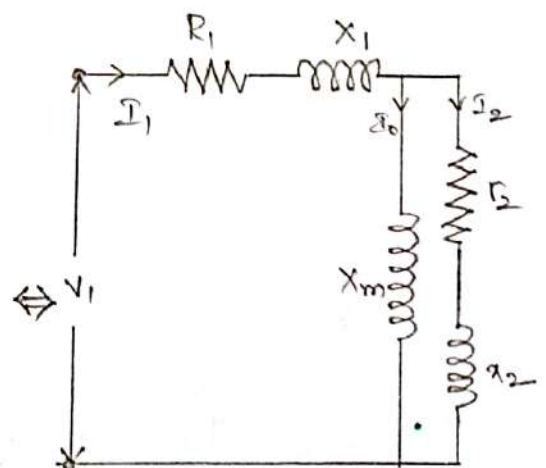
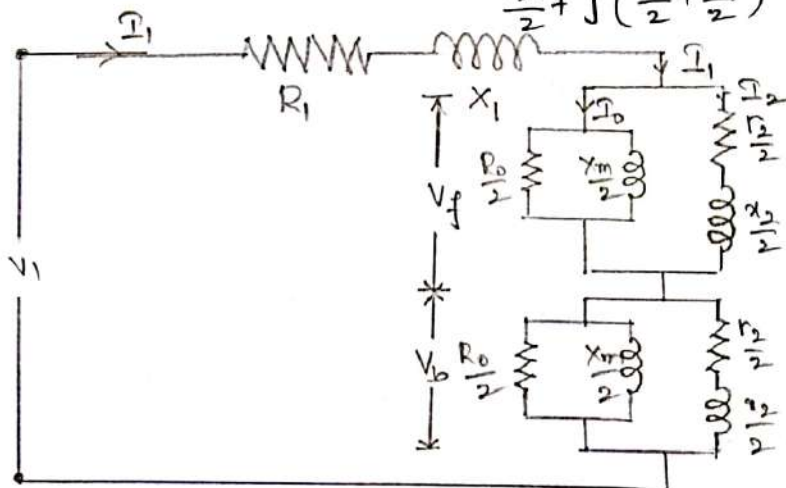
A Single-phase motor may be looked upon as consisting of two motors, having a Common stator winding, but with their respective rotors revolving in opposite directions. The equivalent circuit of such a motor based on double-field revolving theory. Here, the Single phase IM has been imagined to be made up of (i) One stator winding and (ii) two imaginary rotors. The Stator impedance is $Z_1 = R_1 + jX_1$. The impedance of rotor is $(r_2 + jx_2)$, Where r_2 and x_2 are rotor resistance and leakage reactance. R_0 and X_m are core resistance and magnetising reactance. R_0 is neglected ($\because$ very high value, current choose low resistance path, No current flows through the R_0 , Since iron losses has been neglected).

The impedance of forward motor in standstill is

$$Z_{fst} = \frac{j \frac{X_m}{2} \left(\frac{r_2}{2} + j \frac{x_2}{2} \right)}{\frac{r_2}{2} + j \left(\frac{x_2}{2} + \frac{X_m}{2} \right)}$$

The impedance of backward motor in standstill is

$$Z_{bkt} = \frac{j \frac{X_m}{2} \left(\frac{r_2}{2} + j \frac{x_2}{2} \right)}{\frac{r_2}{2} + j \left(\frac{x_2}{2} + \frac{X_m}{2} \right)}$$



R_0 - neglected

The above circuit represents equivalent circuit of a Single-phase Induction Motor at Standstill condition.

The impedance of forward running motor is

$$Z_f = \frac{j \frac{X_m}{2} \left(\frac{r_2}{2s} + j \frac{x_2}{2} \right)}{\frac{r_2}{2s} + j \left(\frac{x_2}{2} + \frac{X_m}{2} \right)}$$

Here $s \rightarrow$ forward slip $= \frac{N_s - N_r}{N_s} = 1 - \frac{N_r}{N_s}$

Backward slip $= \frac{N_s - (-N_r)}{N_s}$ ($\because$ motor revolves opposite direction)

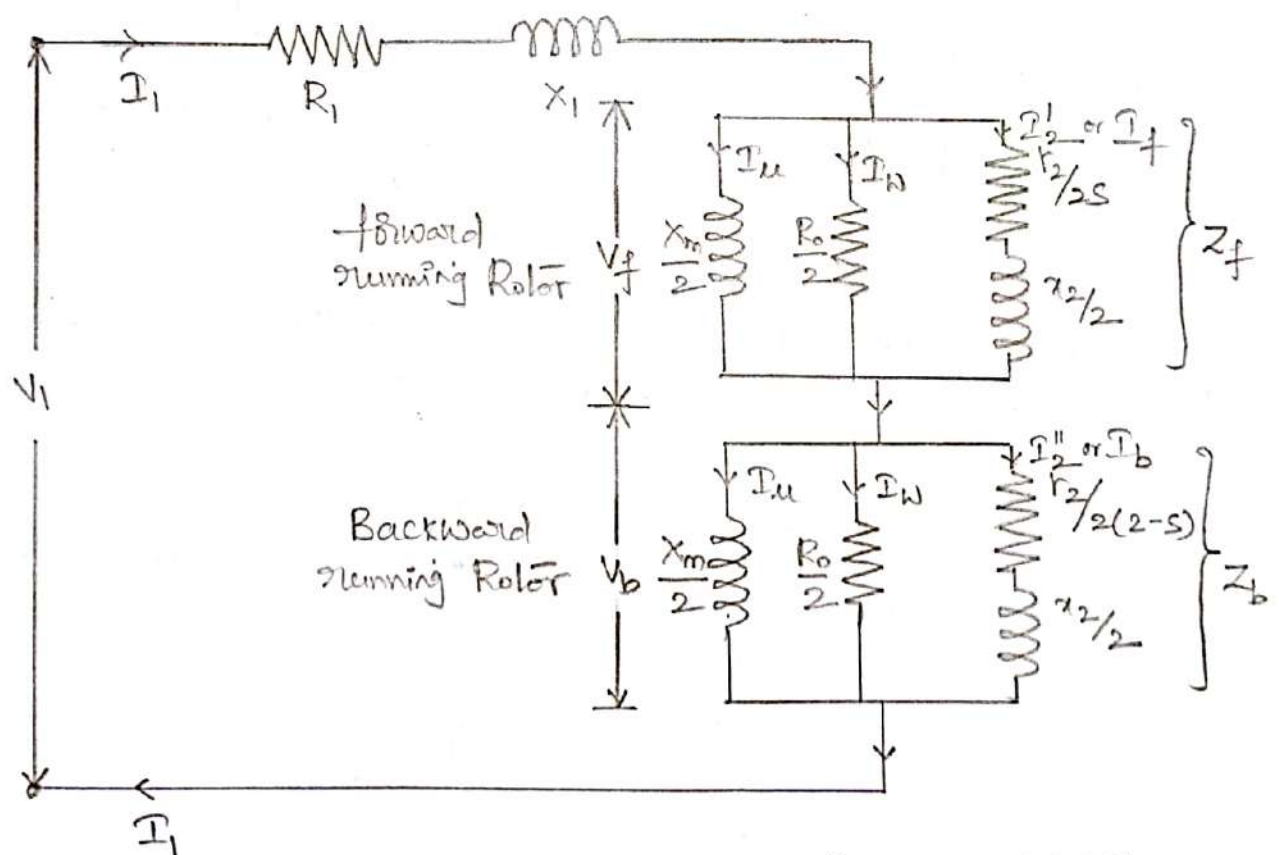
$$= \frac{N_s + N_r}{N_s} = 1 + \frac{N_r}{N_s}$$

$$\text{Backward Slip} = 1 + (1 - s) = (2 - s)$$

The impedance of Backward running motor is

$$Z_b = \frac{j \frac{X_m}{2} \left(\frac{r_2}{2(2-s)} + j \frac{x_2}{2} \right)}{\frac{r_2}{2(2-s)} + j \left(\frac{x_2}{2} + \frac{X_m}{2} \right)}$$

The equivalent circuit of a Single-phase Induction motor in running condition is shown in below.



• Equivalent circuit of 1- ϕ Single phase Induction Motor

The forward torque in synchronous watts is $T_f = I_f^2 r_2 / ks$

The Backward torque in synchronous watts is $T_b = I_b^2 r_2 / 2(2-s)$.

The total torque is $T = T_f - T_b$

Prob: Discuss the revolving field theory of single-phase induction motors. Find the mechanical power output at a slip of 0.05 of the 185W, 4-pole, 110V, 60Hz single phase induction motor, whose constants are given below:

Resistance of the stator main winding, $R_1 = 1.86 \Omega$

Reactance of the stator main winding, $X_1 = 2.56 \Omega$

Magnetizing reactance of the stator main winding, $X_m = 53.5 \Omega$

Rotor Resistance at standstill, $R_2 = 3.56 \Omega$

Rotor Reactance at standstill, $X_2 = 2.56 \Omega$

Sol: Given $R_1 = 1.86 \Omega$ $r_{2/2} = 1.78 \Omega$ $s = 0.05 = 5\%$

$X_1 = 2.56 \Omega$ $x_{2/2} = 1.28 \Omega$

$\frac{X_m}{2} = 26.75 \Omega$

Forward running impedance, $Z_f = \frac{j \frac{X_m}{2} \left[\frac{r_2}{2s} + j \frac{x_2}{2} \right]}{\frac{r_2}{2s} + j \left[\frac{x_2}{2} + \frac{X_m}{2} \right]} = \frac{j 26.75 [1.78/0.05 + j 1.28]}{\frac{1.78}{0.05} + j [1.28 + 26.75]}$

$$Z_f = (11.66 + j 17.36) \Omega = 20.92 \angle 56^\circ \Omega$$

Backward running impedance, $Z_b = \frac{j \frac{X_m}{2} \left[\frac{r_2}{2(2-s)} + j \frac{x_2}{2} \right]}{\frac{r_2}{2(2-s)} + j \left[\frac{x_2}{2} + \frac{X_m}{2} \right]} = \frac{j 26.75 \left[\frac{1.78}{1.95} + j 1.28 \right]}{\frac{1.78}{1.95} + j [1.28 + 26.75]}$

$$Z_b = (0.83 + j 1.2) \Omega = 1.5 \angle 56.3^\circ \Omega$$

Total Impedance, $Z = Z_1 + Z_f + Z_b = (1.86 + j 2.56) + (11.66 + j 17.36) + (0.8 + j 1.2)$
 $= (14.35 + j 21.22) \Omega = 25.53 \angle 55.8^\circ \Omega$

Motor current, $I_1 = \frac{V_1}{Z} = \frac{110}{25.53} = 4.3 A$

Forward Running Voltage, $V_f = I_1 Z_f = 89.9 V$

Backward Running Voltage, $V_b = I_1 Z_b = 6.45 V$

Forward Running current, $I_f = \frac{V_f}{\sqrt{\left(\frac{r_2}{2s}\right)^2 + \left(\frac{x_2}{2}\right)^2}} = 2.5 A$

Backward Running current, $I_b = \frac{V_b}{\sqrt{\left(\frac{r_2}{2(2-s)}\right)^2 + \left(\frac{x_2}{2}\right)^2}} = 4.2 A$

$$\text{Torque [forward running]} = T_f = \frac{I_f^2 r_2}{2s} = 222.5 \text{ Sync. Watt}$$

$$\text{Torque [Backward running]} = T_b = \frac{I_b^2 r_2}{2(2-s)} = 16.10 \text{ Sync. Watt}$$

$$\text{Torque developed, } T = T_f - T_b = 206.4 \text{ Sync. Watt}$$

$$\text{Output} = \text{Sync. Watt} \times (1-s) = 196 \text{ W}$$

Prob: Find the mechanical power output of 185 W, 4-pole, 110 V, 50 Hz Single phase induction motor, whose constants are given below at a slip of 0.05.

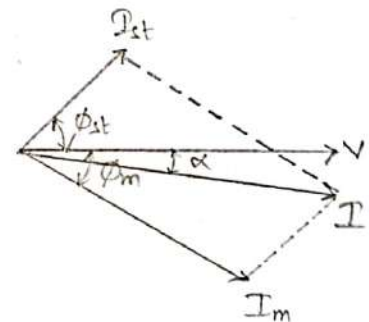
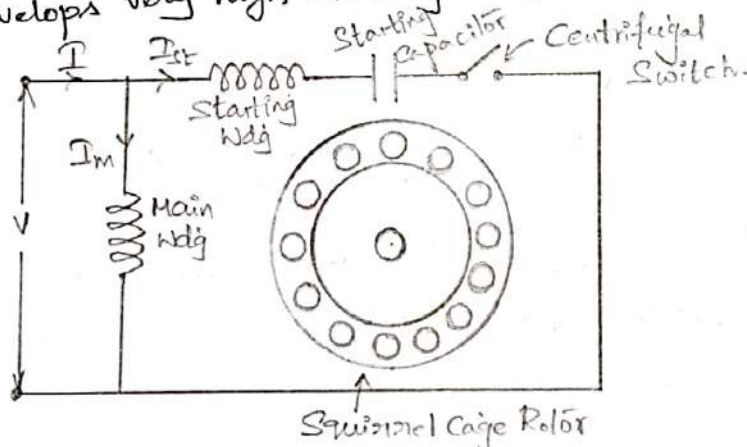
$$R_1 = 1.86 \Omega, X_1 = 2.56 \Omega, X_m = 53.5 \Omega, R_2 = 3.56 \Omega, X_2 = 2.56 \Omega$$

$$\text{Core loss} = 3.5 \text{ W, Friction \& Windage loss} = 13.5 \text{ W. [Ans: 181.5 W]}$$

* Capacitor Start Motor:

In this motor also, the main and auxiliary windings are space displaced by 90° . The time displacement between the currents in the main and auxiliary windings is achieved by connecting a capacitor in series with auxiliary winding as shown in figure. The feature of the capacitive circuit is to draw the leading current.

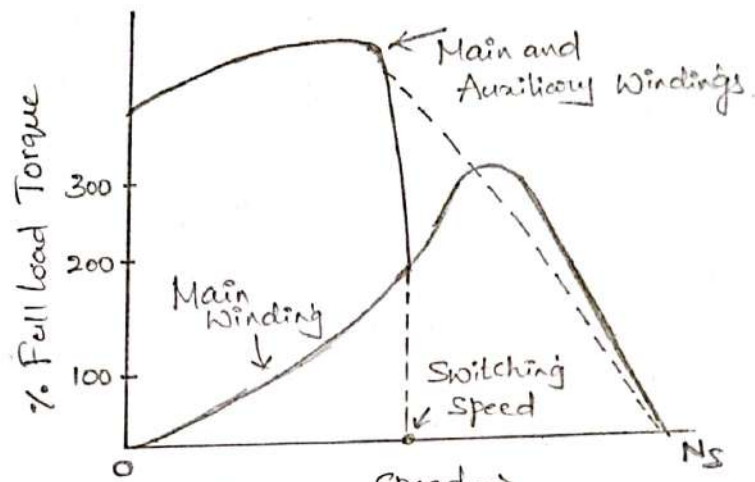
The current drawn by the main winding lags voltage by angle ϕ_m while current drawn by the capacitive start winding leads voltage by angle ϕ_{st} . Hence there exists a large phase difference between I_m and I_{st} which is almost 90° shown in phasor diagram. Thus the motor develops very high starting torque.



Both the capacitor and starting windings are automatically disconnected by centrifugal switch, when the motor has reached 75% of synchronous speed. Because the capacitor is in the circuit only during the starting period it can be an electrolytic type.

The capacitor value can be selected as per the requirement

of starting torque, the starting torque can be as high as 350 to 400% of full load torque. The torque-speed characteristics are shown below.

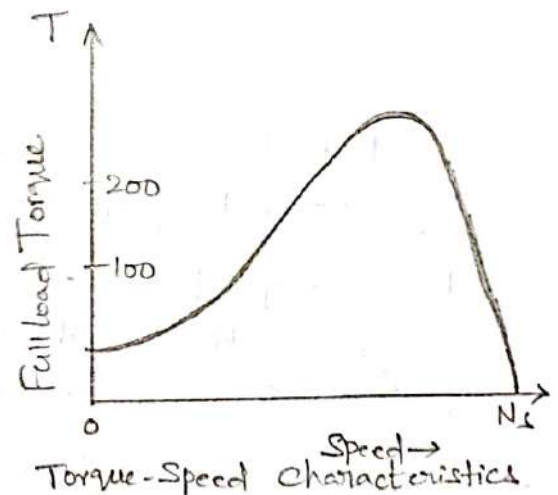
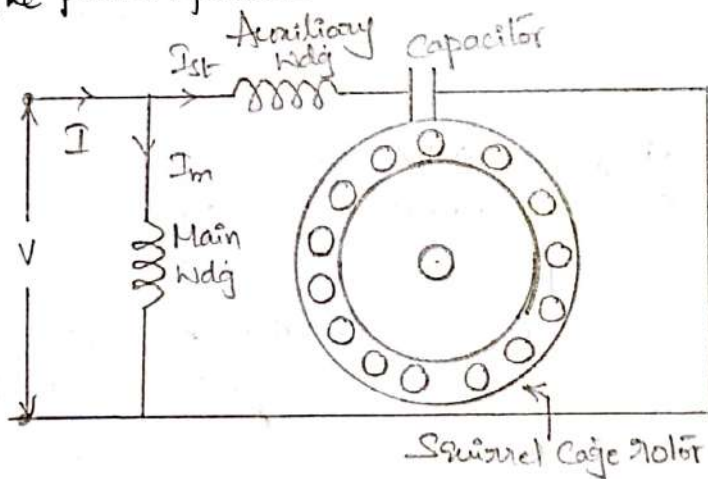


Speed Reversal: The direction of rotation can be changed by interchanging the connections of either the main winding or auxiliary winding terminals.

Applications: These motors are used where high starting torque is required such as in compressors, pumps, air conditioners, conveyors etc.

* Capacitor Motor (or) Permanent Split Capacitor Motor:

In this motor, there is no centrifugal switch and capacitor remain in the circuit during starting as well as running conditions. This improves the power factor.



The performance at starting and running conditions depends on the capacitor 'C', hence its value is to be designed so as to compromise between best starting and the best running conditions. Hence, the starting torque of the motor is about 50 to 100% of the full load torque.

The main advantages of the leaving the capacitor in the circuit are:

1. Higher Power factor
2. Higher Efficiency

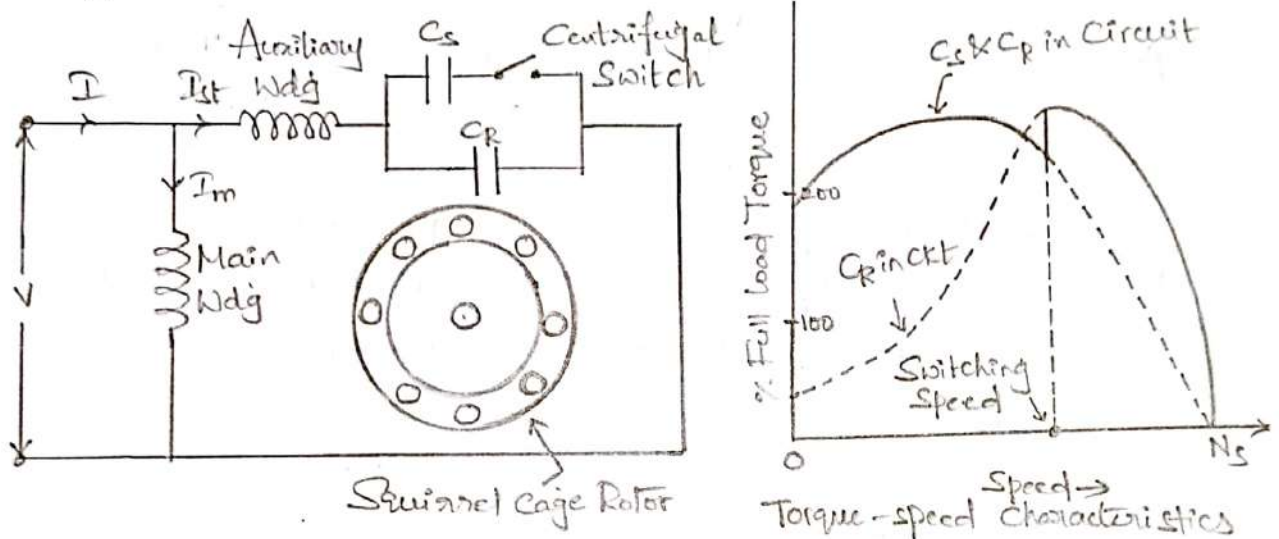
3. Increased over load Capacity of the motor and
4. Noticeless running of the motor.

Speed Reversal: The direction of rotation can be changed by interchanging the connections of either the main winding or auxiliary windings terminals.

Applications: These motors are used for applications requiring low starting torque such as fans, blowers, oil burners etc.

* Capacitor Start Capacitor Run Motor: (Two value Capacitor Motor):

To increase the starting torque, two capacitors are used in the auxiliary winding. Theoretically, optimum starting and running performance can be achieved by having two capacitors, one for starting and other for running as shown in figure(a).

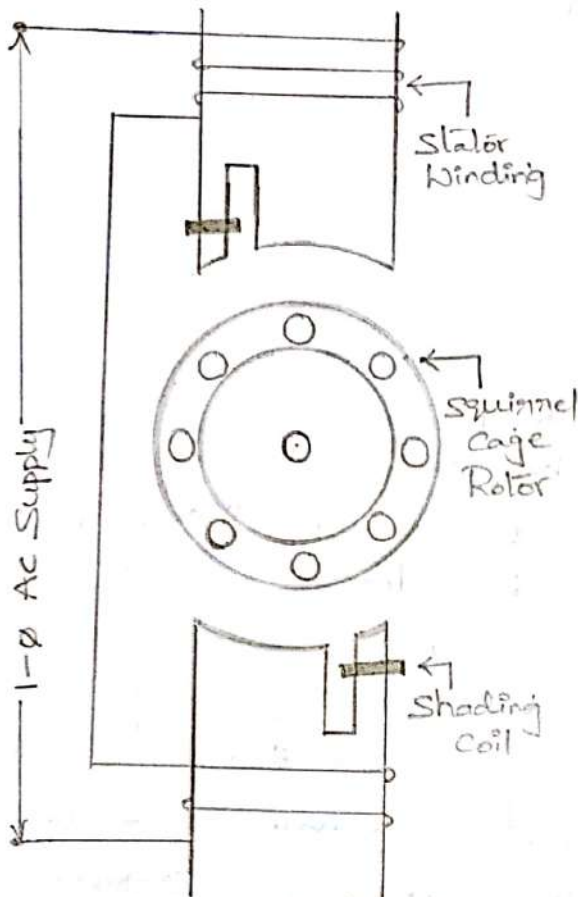


Both the capacitors C_s and C_R are in the circuit during starting. After the motor picks up the speed, the centrifugal switch disconnects the starting capacitor, C_s . The auxiliary winding and running capacitor, C_R , remain in the circuit during running condition also. The capacitor, C_s , is larger in value and is of electrolytic type, permits high starting torque. The capacitor C_R is smaller in value and is of oil type. The motor is more expensive than a capacitor start motor however it provides the best performance.

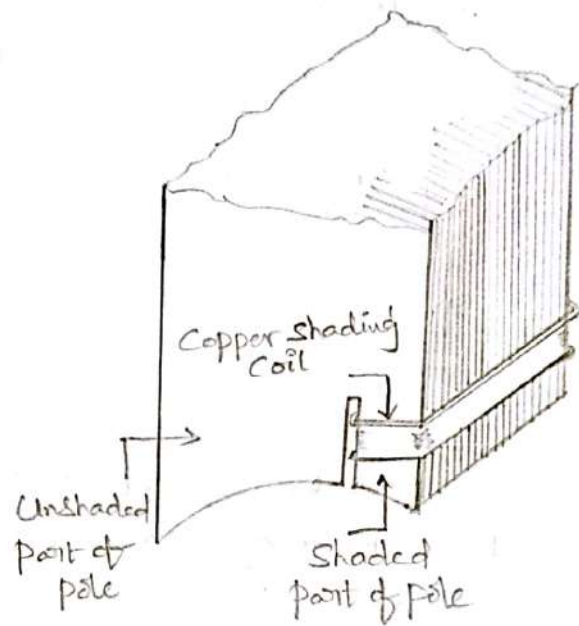
Applications: These are used in compressors, conveyors, pumps & other high torque loads.

* Shaded Pole Motor:

This type of motor consists of a squirrel cage motor and stator consists of salient poles i.e., projected poles. The poles are shaded i.e., each pole carries a short circuited copper coil known as shading coil or band. portion of pole carries a shading band is called as shaded part of the pole and the portion is called as unshaded part of pole.

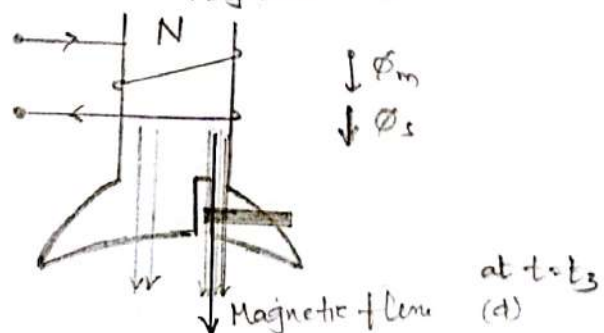
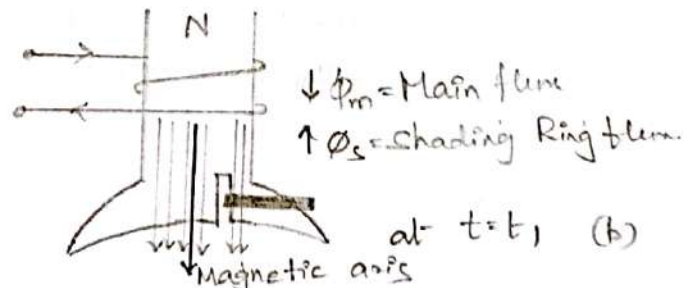
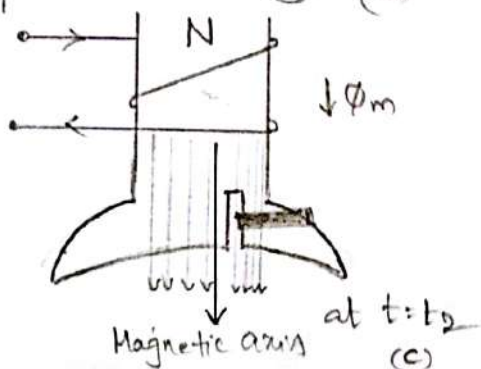
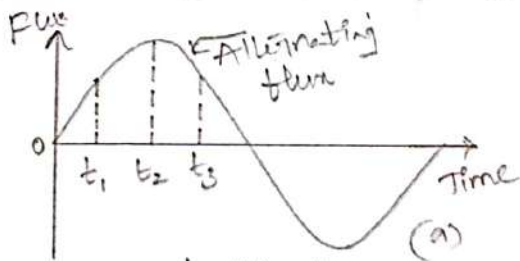


Shaded pole Construction



When 1- ϕ ac supply as given to the stator winding due to shading provided to the poles, a rotating magnetic field is generated. The production of rotating magnetic field can be explained as follows:

The current carried by the winding is alternating and produces alternating flux. The wave-form of a flux shown in below figure (a). Consider the three instants say t_1, t_2 & t_3 during positive half cycle of the flux.

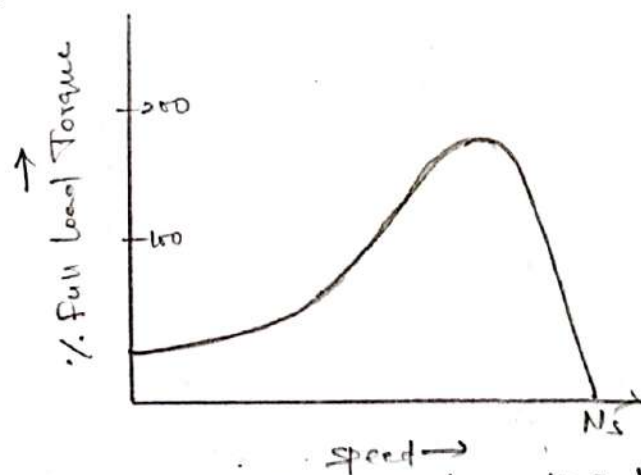


- (1) At instant $t=t_1$, the current in the coil and the flux is increasing. Since there is rate of change of flux, emf will be induced in the shading band. This circulates current through the shading band as it is short circuited producing its own flux. According to lenz's law, shading ring flux will now oppose the cause which is increase in flux so it opposes the main flux. Hence there is crowding of flux in unshaded part while weakening of flux in shaded part. Hence magnetic axis will be at the center of the unshaded poles [in fig (b)].
- (2) At instant $t=t_2$, the current and the flux is reached to its maximum value. So rate of change of flux is minimum. The induced emf in shading coil is very small hence opposition of shading ring flux will be negligible. The main flux distribution is therefore uniform and the magnetic axis is at the center of the pole [in fig (c)].
- (3) At instant $t=t_3$, the current and the flux decreasing. Since there is rate of change of flux, emf will be induced in the shaded ring. The flux produced by current in the shaded ring will now oppose the cause which is decrease in flux. So its direction is same as that of main flux, strengthening it. So there is crowding of flux in the shaded part as compared to unshaded part. Due to this magnetic axis shifts to the center of the shaded pole. [in Fig (d)].

This sequence keeps on repeating for negative half cycle also.

Consequently this produces an effect of rotating field moving in the direction from the unshaded to the shaded part of the pole. Due to this motor produces the starting torque and starts rotating.

The starting torque is very low which is about 40 to 50% of the full load torque. The size & power rating of these motors is very small. These are usually available in a range of $1/300$ to $1/20$ HP.



Speed-Torque characteristics.

Advantages:

- (1) Simple Construction
- (2) Cheap
- (3) Extremely Rugged and
- (4) No Centrifugal Switch is required.

Disadvantages:

- (1) Starting torque is poor.
- (2) Efficiency is very low due to copper losses in the shading ring and
- (3) Very limited overload capacity.

Speed Reversal: The Speed reversal is very difficult. To reverse the direction of rotation, two sets of shading rings are to be provided on both portions of the poles. By opening one of the ring and closing the other ring particular direction of rotation can be achieved. But the method is complicated and expensive.

Applications: These are used in small fans, toys, hair dryers, film projectors, advertising displays etc., where starting torque required is very low.

* Speed Control:

These motors are used only in small sizes, the only economical method is to change the voltage applied to stator. This can be achieved in any of the following ways:

1. Use of auto-transformer. This method is expensive.
2. SCR or Triac circuit may be used to reduce the voltage by ac phase control.
3. A resistor in series with motor circuit gives a very cheap method of voltage control. It has the disadvantage of reduced efficiency due to energy loss in the resistor.

* Single-Phase A.C. Series Motor:

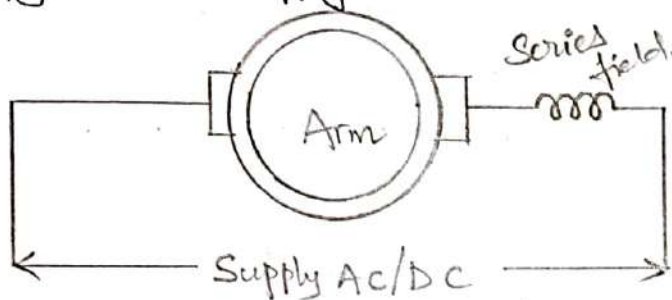
When normal d.c. series motor is connected to an a.c. supply, both field and armature currents reverse simultaneously and unidirectional torque is produced in the motor.

Consider the case of a two pole motor and let the alternating current be in its positive half, then the polarity of the field poles and the currents flowing through the armature conductors be as indicated in fig (2). The armature conductors carry inward currents $\otimes$ under N-pole and outward currents $\odot$ under S-pole. By applying Fleming's

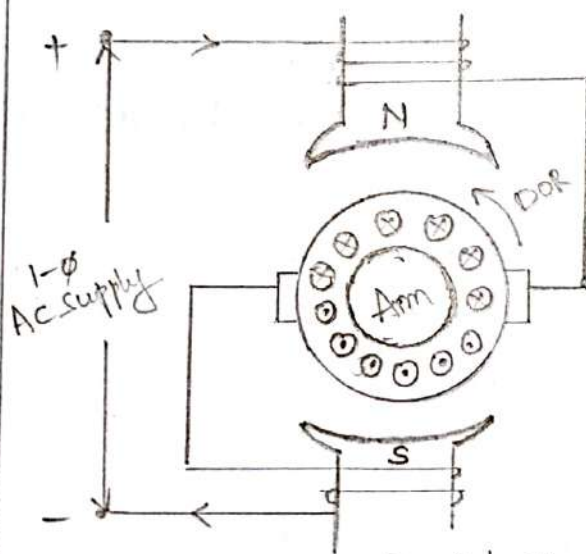
left hand rule it will be seen that the torque developed in the armature will try to rotate in anti-clockwise direction.

During the next instant, the alternating current goes through the negative half cycle. Now the current through the field wdg and armature will also change as shown in fig(3). It will be again seen that the armature will tend to rotate in the same direction because of uniform torque produce the two halves of the cycle.

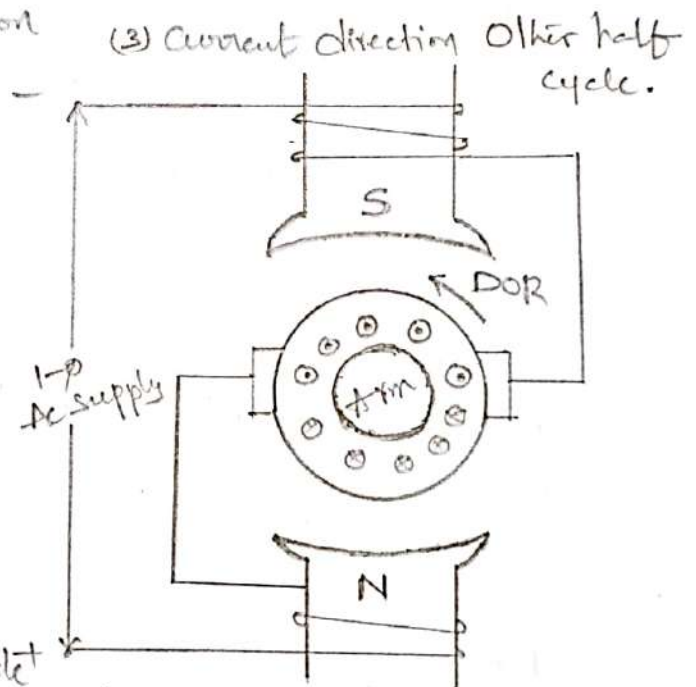
Thus a series motor has the ability to run on both the d.c. supply and a.c. supply.



(1) Schematic Representation



(2) Current Directions First half cycle



(3) Current directions Other half cycle.

The performance of D.C. Series motor works on A.C. supply is not satisfactory due to the following reasons:

1. The efficiency is low. This is because of the increase in core losses due to alternating flux.
2. Armature & field winding offer high reactance to a.c. due to which operating power factor is very poor.
3. Considerable sparking at brushes will occur. This is due to poor

Commutation. The voltage induced by transformer action in the coil undergoing commutation further intensifies commutation difficulties.

Modifications in Design of A.C. Series Motor:

Some modifications are required to have the satisfactory performance of d.c. series motor on a.c. supply, when it is called as a.c. series motor. The modifications are:

1. To reduce the eddy current losses, yoke and poles should be fully laminated.
2. The power factor can be improved by reducing field and armature reactances. [The field wdg is designed with less no. of turns].
3. The motor should be provided with a large no. of poles each supplying less flux per pole.
4. Reduction in no. of turns on the field winding would also reduce the field flux. To keep the torque same, it is necessary to increase the armature turns proportionately. This increases the armature reaction and armature reactance.
5. Compensating wdg should be employed to lower the armature reactance as far as possible. Compensation also improves the commutation.
6. The armature coils are single turn coils and brushes of less width are used not to short circuit more than two coils at a time.
7. The air gap is made very small so that less field turns can be used per pole.
8. The frequency of supply used is reduced. The transformer emf is proportional to frequency and hence good commutation is easy at lower frequencies.

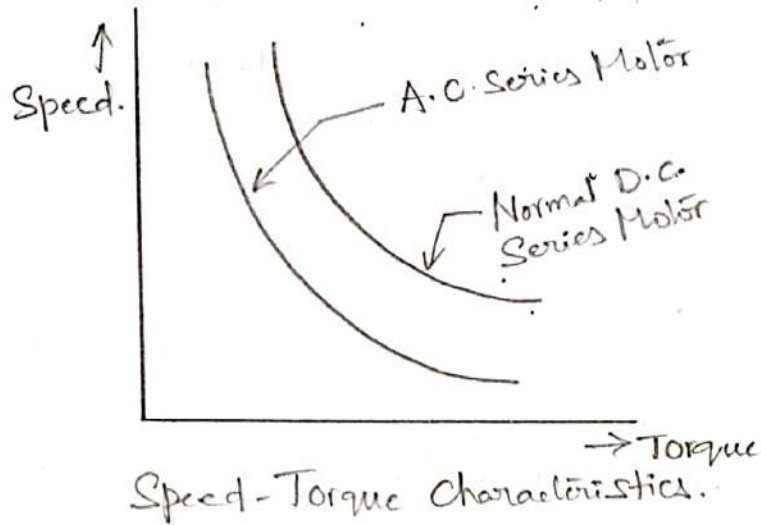
Speed Control:

Speed regulation is easily obtained through a change of supply voltage to the a.c. series motor by means of a tapped transformer.

Characteristics:

The characteristics of a.c. series motor are similar to that of d.c. series motor. The torque varies as square of the armature current and speed varies inversely as the armature current. The speed

of such motor can be dangerously high on no-load condition. and hence it is always started with some load. Starting torque produced is high which is 3 to 4 times the full load torque.



Applications:

Because of high starting torque it is used in electric traction, hoists, locomotives etc.

Synchronous Generators (Alternators)

In D.C. generators, the field poles are stationary and the armature conductors rotate. The voltage generated in the armature conductors is of alternating nature. This generated alternating voltage is converted to direct voltage at the brushes with the help of the Commutator.

Ac generators are usually called alternators. They are also called Synchronous generators. Rotating machines that rotate at a speed fixed by the supply frequency and the number of poles are called Synchronous machines.

A Synchronous generator is a machine for converting mechanical power from a prime mover to a.c. electric power at a specific voltage and frequency. A synchronous machine rotates at a constant speed called Synchronous speed. A Synchronous motor is a synchronous machine that converts electric power to mechanical power. Synchronous generators are usually of 3- ϕ type because of the several advantages of 3-ph generation, transmission and distribution.

Large Synchronous generators are used to generate bulk power at thermal, hydro and nuclear power stations. The biggest size used in India has a rating of 500MVA used in Super-thermal power stations. For bulk power generation, stator windings of synchronous generators are designed for voltages ranging from 6.6 kV to 33 kV.

* Advantages of Rotating field Alternator:

Most alternators have the rotating field and the stationary armature. The rotating field type alternator has several advantages over the rotating armature type alternator:

- (i) A stationary armature is more easily insulated for the high voltage for which the alternator is designed.
- (ii) The armature windings, being stationary, are not subjected to vibration and centrifugal forces.
- (iii) The output current can be taken directly from fixed terminals on the stationary armature without using slip rings, brushes etc.
- (iv) The rotating field is supplied with direct current. Usually the field

Voltage is between 100 to 500 volts. Only two slip rings are required to provide direct current for the rotating field, while at least three slip rings would be required for a rotating armature.

- (v) The bulk and weight of the armature windings are substantially greater than the windings of the field poles. The size of the machine is, therefore, reduced.
- (vi) Rotating field is comparatively light and can be constructed for high speed rotation. The armatures of large alternators are forced cooled with circulating gas or liquids.
- (vii) The stationary armature may be cooled more easily because the armature can be made large to provide a number of cooling ducts.

* Speed and Frequency:

The frequency of the generated voltage depends upon the no. of field poles and on the speed at which field poles are rotated. One complete cycle of voltage is generated in an armature coil when a pair of field poles passes over the coil.

Let P = total no. of poles

p = pair of field poles = $\frac{P}{2}$

N = Speed of the field poles in rpm

n = Speed of the field poles in rps = $\frac{N}{60}$

f = frequency of the generated voltage in Hz.

In one revolution of the rotor, an armature coil is cut by $\frac{P}{2}$ north pole and $\frac{P}{2}$ south pole. Since one cycle is generated in an armature coil when a pair of field poles passes over the coil, the no. of cycles generated in one revolution of the rotor will be equal to the no. of pair of poles. That is,

no. of cycles per revolution = p

no. of revolutions per second = n

$\therefore$ frequency = no. of cycles per second

= no. of cycles per revolution $\times$ no. of revolutions per sec

= $p \times n$

$$f = \frac{P}{2} \times \frac{N}{60} \Rightarrow f = \frac{PN}{60 \times 2} \Rightarrow$$

$$N = \frac{120f}{P}$$

Synchronous Speed:

The Synchronous Speed, $N_s = \frac{120f}{P}$

A machine which runs at synchronous speed is called synchronous machine.

No. of poles	Synchronous Speed, N_s (rpm).
2	3000
4	1500
6	1000
8	750
10	600
12	500

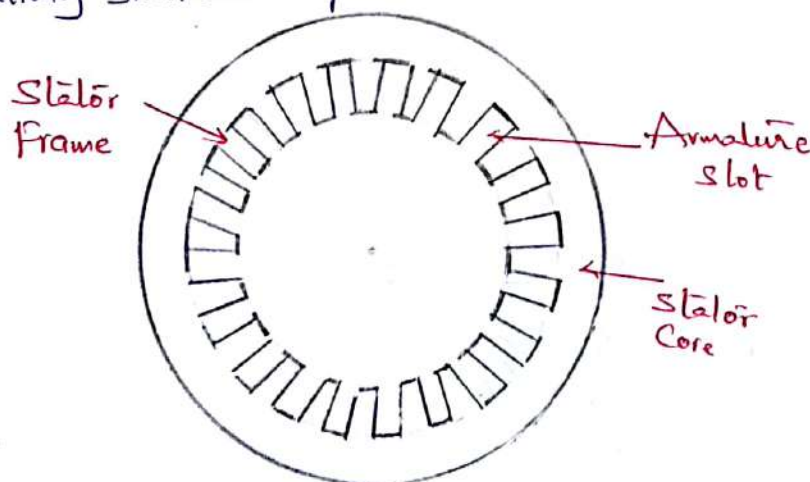
* Construction of Three-phase Synchronous Machines:

An Alternator consists of two main parts

- (i) Stator (Stationary part)
- (ii) Rotor (Rotating part)

(i) Stator:

The various parts of the stator include the frame, stator core, stator windings and cooling arrangement. The frame may be of cast iron for small size machines and of welded steel type for large size machines. In order to reduce hysteresis and eddy-current losses, the stator core is assembled with high grade silicon content steel laminations. A 3- ϕ windings is put in the slots cut on the inner periphery of the stator as shown below. The winding is star connected. The winding of each phase is distributed over several slots. When current flows in a distributed windings it produces an essentially sinusoidal space distributed emf.



(ii) Rotor:

There are two types of rotor constructions, namely

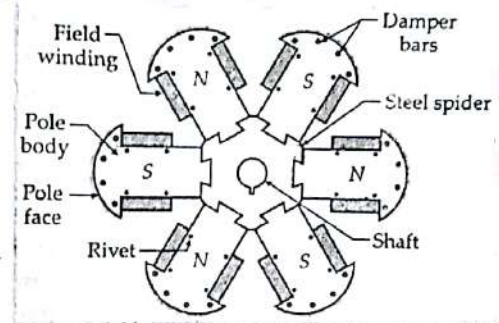
- (i) Salient - pole type (or) Projected pole type
- (ii) cylindrical type (or) phase wound type

Salient Pole Rotor:

The term Salient means "protruding" or "Projecting". Thus, a salient-pole rotor consists of poles projecting out from the surface of the rotor core. Salient pole rotors are normally used for rotors with four or more poles.

Since the rotor is subjected to changing magnetic fields, it is made of thin steel laminations to reduce eddy current losses.

Salient pole rotors have concentrated winding on the poles. Damper bars are usually inserted in the pole faces to damp out the rotor oscillations during sudden change in load conditions.



[fig. Six pole Salient pole rotor]

A salient pole synchronous machine has a non-uniform air gap. The air gap is minimum under the pole centers and it is maximum in between the poles. The pole faces are so shaped that the radial air gap length increases from the pole center to the pole tips so that the flux distribution in the air gaps is sinusoidal. This will help the machine to generate sinusoidal emf.

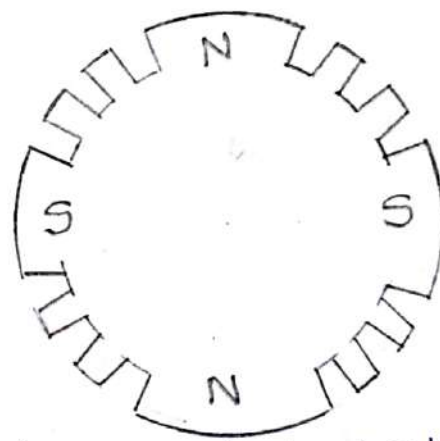
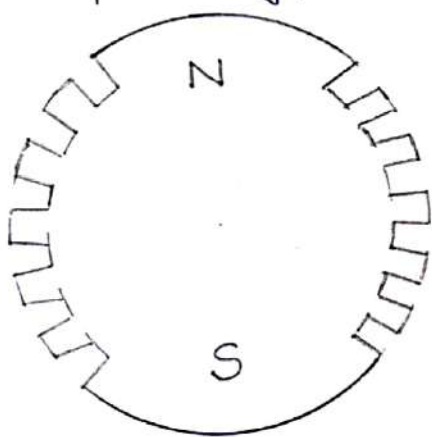
The individual field pole-pole windings are connected in series to give alternate north and south polarities. The end of the field windings are connected to a d.c. source (a d.c. generator or a rectifier) through the brushes on the slip rings. The slip rings are metal rings mounted on the shaft and insulated from it. They are used to carry current to or from the rotating part of the machine via carbon brushes.

Salient pole generators have a large no. of poles, and operate at lower speeds. A salient pole generator has comparatively a large diameter and a short axial length. The large diameter accommodates a large number of poles.

Salient pole alternators driven by water turbines are called hydro-alternators or Hydro generators.

Cylindrical Rotor:

A cylindrical rotor machine is also called non-salient pole rotor machine. It has its rotor so constructed that it forms a smooth cylinder. Cylindrical rotors are made from solid forgings of high grade nickel-chrome-molybdenum steel. In about $2/3$ of the rotor periphery, slots are cut at regular intervals and parallel to the shaft. The d.c. field windings are accommodated in these slots. The winding is of distributed type. The unslotted portion of the rotor forms two (or four) pole faces. A cylindrical rotor machine has a comparatively small diameter and long axial length. Such a construction limits the centrifugal forces. Thus, cylindrical rotors are particularly useful in high-speed machines. The smooth rotor of the machine makes less winding losses and the operation is less noisy because of uniform air gap.



[Fig. End view of two-pole & four-pole cylindrical rotors]

Cylindrical rotor machines are driven by steam or gas turbines. Cylindrical rotor synchronous generators are called turbo alternators or turbo generators. The biggest size used in India has a rating of 800 MVA installed in super thermal power plants.

* Excitation Systems for Synchronous Machines:

Excitation means production of flux by passing current in the field winding.

Direct current is required to excite the field winding on the rotor of the sync. generator. For small machines, dc is supplied to the rotor field by d.c. generator is called exciter. This exciter may be supplied current by a smaller d.c. generator called pilot exciter. The main and pilot

exciters are mounted on the main shaft of the synchronous machine. The d.c. output of the main exciter is given to the field winding of the synchronous machine through brushes and slip rings.

For medium size machines a.c. exciters are used in place of d.c. exciters. A.c. exciters are three phase a.c. generators. The output of an a.c. exciter is rectified and supplied through brushes and slip rings to the rotor winding of the main synchronous machine.

For large synchronous generators with ratings of the order of few hundred MW, the excitation requirements become very large. The problem of conveying such amounts of power through high-speed sliding contacts becomes formidable. At present large synchronous generators and synchronous motors are using brushless excitation systems. A brushless exciter is a small direct-coupled a.c. generator with its field circuit on the stator and the armature circuit on the rotor. The three phase output of the a.c. exciter generator is rectified by solid state rectifiers. The rectifier output is connected directly to the field winding, thus eliminating the use of brushes and slip rings.

A brushless excitation system requires less maintenance due to absence of brushes and slip rings. The power loss is also reduced.

* EMF Equation of an Alternator:

Let ϕ = useful flux per pole in wb

P = total no. of poles

Z_p = total no. of conductors or coil sides in series per phase

T_p = total no. of coils or turns per phase

n = Speed of rotation of rotor in rps

f = frequency of generated voltage (Hz).

Since the flux per pole is ϕ , each stator conductor cuts a flux

$P\phi$
The average value of generated voltage per conductor

$$= \frac{\text{flux cut per revolution in wb}}{\text{time taken for one revolution in sec}}$$

$$= \frac{P\phi}{1/n} = n\phi P \text{ Volts} \quad \text{--- (1)}$$

We know that $f = \frac{PN}{120} = \frac{P\omega}{2}$

$$\omega_n = 2f$$

$$\therefore \textcircled{1} \Rightarrow E_{av}/\text{Conductor} = 2f\phi$$

Since there are Z_p conductors in series per phase, the average voltage generated per phase is given by

$$E_{av}/\text{phase} = 2f\phi Z_p$$

Since one turn or coil has two sides, $Z_p = 2T_p$

$$\therefore E_{av}/\text{phase} = 4f\phi T_p \text{ --- } \textcircled{2}$$

For the voltage wave, the form factor is given by

$$K_f = \frac{\text{rms value}}{\text{average value}} = 1.11 \text{ (for Sinusoidal Voltage)}$$

Therefore, the rms value of the generated voltage per phase, E_p

$$\begin{aligned} E_p = E_{rms}/\text{phase} &= K_f \times E_{av}/\text{phase} \\ &= 1.11 \times 4f\phi T_p \\ &= 4.44f\phi T_p \end{aligned}$$

$$E_p = 4.44f\phi T_p \text{ --- } \textcircled{3}$$

The eqⁿ ③ has been derived with the following assumptions:

(a) Coils have got full pitch.

(b) All the conductors are concentrated in one stator slot.

* Armature Winding:

The winding through which a current is passed to produce the main flux is called the "field winding". The winding in which voltage is induced is called the "armature winding".

For a p -pole machine, electrical degree is defined as follows:

$$\theta_{elec} = \frac{p}{2} \theta_{mech}$$

The angular distance between the centres of two adjacent poles on a machine is known as "pole pitch" or "pole span".

$$\text{One pole pitch} = 180^\circ_{elec} = \frac{360^\circ_{mech}}{p}$$

The two sides of a coil are placed in two slots on the stator surface. The distance between the two sides of a coil is called the "coil pitch". If the coil pitch is one pole pitch, it is called the "full-pitch".

Coil. If the coil pitch is less than one pole pitch, the coil is called "short pitch coil" or "fractional pitch coil". (Chorded coil).

Coil Span factor or pitch factor:

If the span of the coil is reduced by an angle α elec. degree, the coil span will be $(180^\circ - \alpha)$ elec. degree.

In case of a full-pitch coil, the two coil sides span a distance exactly equal to the pole pitch of 180° elec. degree. As a result, the voltage generated in a full-pitch coil is such that the coil-side voltages are in phase as shown in below.

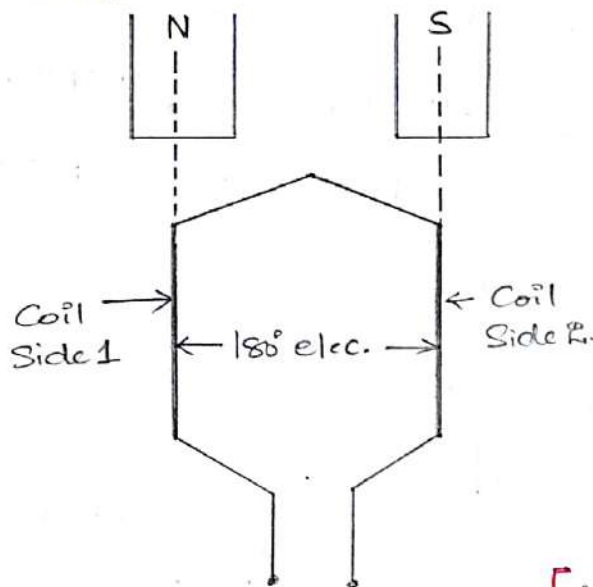
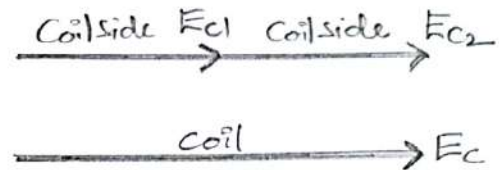


Fig. Full pitch coil



Let E_1 & E_2 be the voltages generated in the coil sides and E_c the resultant coil voltage.

$$\text{Then } E_c = E_1 + E_2$$

$$|E_{c1}| = |E_{c2}| = E_1 \text{ (say).}$$

$$E_c = E_1 + E_2 = 2E_1$$

[$\because E_1$ & E_2 are in phase, The resultant coil voltage E_c is equal to their arithmetic sum.]

If the coil span of a single coil is less than the pole pitch of 180° (elec.), the voltage generated in each coil side are not in phase. The resultant coil voltage E_c is equal to phasor sum of E_{c1} & E_{c2} .

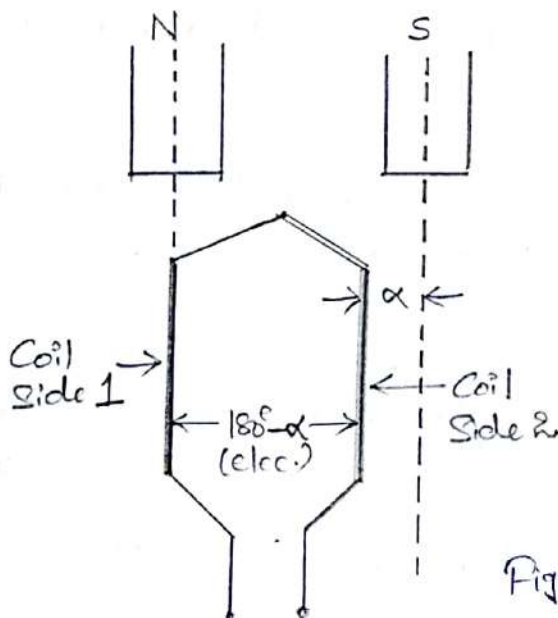
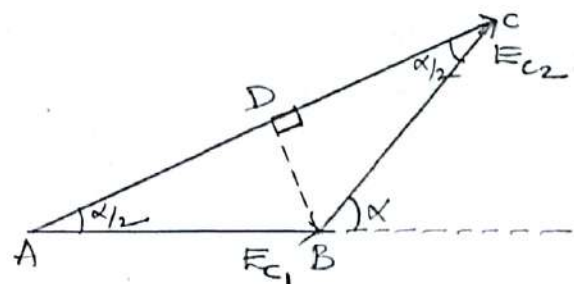


Fig. Short-pitch coil.



If the coil span is reduced by an angle α electrical degree, the coil span is $(180 - \alpha)$ elec. degrees. The voltage generated E_{C1} & E_{C2} in the two coils side will be out of phase with respect to each other by an angle α (elec.) as shown in above figure. The phasor sum of E_{C1} and E_{C2} is $E_C (= AC)$.

The **coil span factor** or **pitch factor** K_c is defined as the ratio of the voltage generated in the short-pitch coil to the voltage generated in the full-pitch coil. The coil span factor is also called the **chording factor**.

$$K_c \triangleq \frac{\text{actual Voltage generated in the coil}}{\text{arithmetic sum of the voltages of two coil sides}} =$$

$$= \frac{AC}{2AB} = \frac{2AD}{2AB} = \cos \frac{\alpha}{2}$$

$$K_c = \cos \left(\frac{\alpha}{2} \right)$$

For full pitch coil, $\alpha = 0^\circ$, $\cos \frac{\alpha}{2} = 1$ & $K_c = 1$. For short pitch coil $K_c < 1$

Advantages of short pitching (or) chording:

- (1) Shortens the ends of the winding and therefore there is a saving in the conductor material.
- (2) Reduces effects on distorting harmonics, and thus the waveform of the generated voltage is improved and making it approach is a sine wave.

Distribution factor (or) Breadth factor (K_d):

In a concentrated winding, the coil sides of a given phase are concentrated in a single slot under a given pole. The individual coil voltages induced in phase with other. These voltages may be added arithmetically. In order to determine the induced voltage per phase, a given coil voltage is multiplied by the number of series connected coils per phase.

In actual practice, in each phase, coils are not concentrated in a single slot, but are distributed in a number of slots in space to form a polar group under each pole. The voltages induced in coil sides constituting a polar group are not in phase but differ by an angle equal to the angular displacement β of the slots. The total voltage

Induced in any phase will be the phasor sum of the individual coil voltages.

The **distribution factor** or **breadth factor** is defined as the ratio of the actual voltage obtained to the possible voltage if all the coils of a polar group were concentrated in a single slot.

$$K_d \triangleq \frac{\text{Phasor Sum of Coil Voltages per phase}}{\text{Arithmetic Sum of Coil Voltages per phase}}$$

Let m = slots per pole per phase, that is slots per phase belt

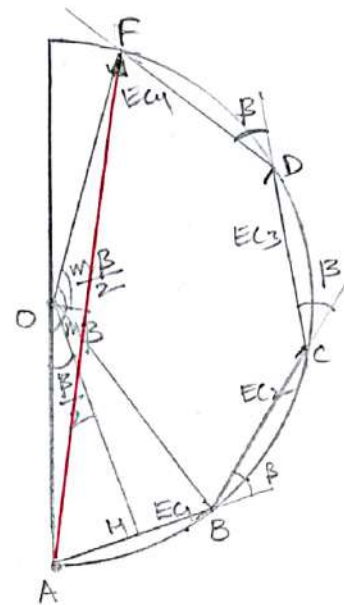
$$m = \frac{\text{Slots}}{\text{poles} \times \text{phases}}$$

β = angular displacement between adjacent slots in elec. degrees.

$$\beta = \frac{180^\circ}{\text{Slots/pole}} = \frac{180^\circ \times \text{pole}}{\text{slots}}$$

Thus, one phase of the winding consists of coils arranged in m consecutive slots. Voltages $E_{c1}, E_{c2}, E_{c3}, \dots$ are the individual coil voltages. Each coil voltage E_c will be out of phase with the next coil voltage by the slot pitch β .

The fig. shows the voltage polygon of the induced voltages in the four coils of a group ($m=4$).



The phasor sum AF , representing the resultant winding voltage, subtends an angle $m\beta$ at the centre.

Arithmetic Sum of individual Coil Voltages

$$= m E_c = m AB = m (2AM)$$

$$= 2m OA \sin \angle AOM = 2m OA \sin (\beta/2)$$

Phasor Sum of individual Coil Voltages

$$= AF = 2AG = 2OA \sin \angle AOG = 2OA \sin \frac{m\beta}{2}$$

$$\therefore K_d = \frac{2OA \sin (m\beta/2)}{2OA m \sin (\beta/2)} \Rightarrow$$

$$K_d = \frac{\sin (m\beta/2)}{m \sin (\beta/2)}$$

It is to be noted that the distribution factor K_d for a given number of phases is dependent only on the number of distributed slots under a given pole. It is independent of the type of the winding, lap or wave, or the number of turns per coil, etc. As the number of slots per pole increases, the distribution factor decreases.

* Actual Voltage generated in Alternators:

Taking the coil span factor and distribution factor into account, the actual generated voltage per phase is given by

$$E_p = 4.44 K_c K_d f \Phi T_p \\ = 2.22 K_c K_d f \Phi Z_p$$

$$K_c \rightarrow \text{Coil Span factor} = \cos(\alpha/2)$$

$$K_d \rightarrow \text{Distribution factor} = \frac{\sin(m\beta/2)}{m \sin(\beta/2)}$$

It is smaller than the actual number of turns per phase due to fractional pitch coils and due to distribution of winding over several slots under each pole.

The coil span factor and distribution factor of a winding are sometimes combined into a single "winding factor K_w " which is the product of $K_c \times K_d$. That is,

$$K_w \triangleq K_c \cdot K_d$$

For star connected alternator, the line voltage is $\sqrt{3}$ times the phase voltage.

$$E_L = \sqrt{3} E_p$$

The angle between the terminal voltage V and the internal voltage E is the "machine angle" or "rotor angle" (δ).

Prob: A 3- ϕ , 50Hz, 8-pole alternator has a star-connected winding with 120 slots and 8 conductors per slot. The flux per pole is 0.05 wb, sinusoidally distributed. Determine the phase & line voltages.

Sol: Given that $f = 50\text{Hz}$

$$p = 8$$

$$Z = 120 \times 8 = 960 \text{ Conductors} \Rightarrow Z_{ph} = \frac{960}{3} = 320$$

$$\phi = 0.05 \text{ wb}$$

Let us take the full-pitch coil

$$\alpha = 0^\circ \Rightarrow \cos \alpha/2 = K_c \Rightarrow K_c = \cos 0^\circ = 1$$

$$m = \frac{\text{Slots}}{\text{Poles} \times \text{phases}} = \frac{120}{8 \times 3} = 5$$

$$\beta = \frac{180^\circ \times \text{Poles}}{\text{Slots}} = \frac{180^\circ \times 8}{120} = 12^\circ$$

$$K_d = \text{Distribution factor} = \frac{\sin(m\beta/2)}{m \sin(\beta/2)} = \frac{\sin(5 \times 12^\circ/2)}{5 \sin(12^\circ/2)} = 0.9567$$

$$\begin{aligned} \text{Generated Voltage/ph} &= 2.22 f \phi K_c K_d Z \\ &= 2.22 \times 50 \times 0.05 \times 1 \times 0.9567 \times 320 \\ &= 1699 \text{ V} \end{aligned}$$

$$\begin{aligned} \text{Generated Voltage (Line)} &= \sqrt{3} \times \text{Generated Voltage/ph} \\ &= \sqrt{3} \times 1699 \\ &= 2942.8 \text{ V} \end{aligned}$$

Prob: A 3- ϕ , 6-pole, star connected alternator revolves at 1000 rpm. The stator has 90 slots and 8 conductors per slot. The flux per pole is 0.05 Wb. (Sinusoidally distributed). Calculate the Voltage generated in the machine if the winding factor is 0.96.

Prob: A 3- ϕ , 50 Hz, 2-pole, star connected turbo alternator has 54 slots with 4 conductors per slot. The pitch of the coils is 2 slots less than the pole pitch. If the machine gives 3300 V between lines on open circuit with sinusoidal flux distribution, determine the useful flux per pole.

Sol: $m = \frac{\text{Slots}}{\text{Poles} \times \text{phases}} = \frac{54}{2 \times 3} = 9$

$$\beta = \frac{180^\circ \times \text{Poles}}{\text{Slots}} = \frac{180^\circ \times 2}{54} = \frac{20^\circ}{3}$$

$$\text{Pole pitch} = \frac{\text{No. of slots}}{\text{No. of poles}} = \frac{54}{2} = 27$$

Coil pitch is 2 slots less than the pole pitch = 25 slots

$$\alpha = 2 \times \frac{20^\circ}{3} = \frac{40^\circ}{3}$$

$$K_c = \cos \frac{\alpha}{2} = \cos \frac{20^\circ}{3} = 0.9932$$

$$K_d = \frac{\sin \frac{m\beta}{2}}{m \sin \beta/2} = \frac{\sin 9 \times \frac{20^\circ}{6}}{9 \sin \frac{20^\circ}{6}} = 0.9554$$

$$\text{No. of conductors per phase, } Z_p = \frac{54 \times 4}{3} = 72$$

$$E_p = 2.22 f \phi K_c K_d Z_p$$

$$\Rightarrow \phi = \frac{E_p}{2.22 f K_c K_d Z_p} = \frac{3300/\sqrt{3}}{2.22 \times 50 \times 0.9932 \times 0.9554 \times 72} = 0.2512 \text{ Wb}$$

Prob. A 4-pole, 3- ϕ , 50 Hz, star connected alternator has 60 slots, with 2 conductors per slot and having armature winding of the two-layer type. Coils are short pitched in such a way that if one coil side lies in slot number 1, the other lies in slot no 13. Determine the useful flux per pole required to generate a line voltage of 6000V.

Sol:

$$\text{Slot angle, } \beta = \frac{180^\circ \times \text{poles}}{\text{slots}} = \frac{180 \times 4}{60} = 12^\circ$$

$$\text{coil span} = 12\beta = 12 \times 12 = 144^\circ$$

$$\alpha = 180^\circ - 144^\circ = 36^\circ$$

$$\text{Coil span factor, } K_c = \cos \frac{\alpha}{2} = \cos \frac{36^\circ}{2} = 0.951$$

$$m = \frac{\text{slots}}{\text{poles} \times \text{phases}} = \frac{60}{4 \times 3} = 5$$

$$\text{Distribution factor, } K_d = \frac{\sin(m\beta/2)}{m \sin(\beta/2)} = \frac{\sin(5 \times 12^\circ/2)}{5 \sin(12^\circ/2)} = 0.9567$$

$$Z_p = \frac{60 \times 2}{3} = 40$$

$$E_p = 2.22 f \phi K_c K_d Z_p$$

$$\Rightarrow \phi = \frac{E_p}{2.22 f K_c K_d Z_p} = \frac{6000/\sqrt{3}}{2.22 \times 50 \times 0.951 \times 0.9567 \times 40} = 0.8576 \text{ wb.}$$

* Armature Reaction in Synchronous Machines:

When current flows through the armature winding of an alternator, the resulting mmf produces flux. The armature flux reacts with the main pole flux, causing the resultant flux to become either less than or more than the original field flux. The effect of armature (stator) flux on the flux produced by the rotor field poles is called "armature reaction".

The reaction in synchronous machines affects the main-field flux very differently for different power factors. Consider the unity power-factor, zero power factor lagging and zero power factor leading for the armature reaction of synchronous machine.

Summary of Nature of Armature Reaction:

From the above discussions, the following more general conclusions regarding the nature of armature reaction can be drawn for a synchronous generator supplying a balanced 3-phase load:

1. The armature reaction flux is constant in magnitude and rotates at synchronous speed.
2. The armature reaction is cross-magnetizing when the generator supplies a load at unity factor (power factor).
3. When the generator supplies a load at lagging power factor, the armature reaction is partly demagnetizing and partly cross-magnetizing.
4. When the generator supplies a load at leading power factor, the armature reaction is partly magnetizing and partly cross-magnetizing.

Armature Reaction in a Motoring Machine:

Since the synchronous machine operating in motoring mode, the armature reaction mmf and flux are in phase opposition, the nature of armature reaction is just the reverse of what is stated for the synchronous generator. The corresponding conclusions for a synchronous machine operating in a motoring mode as follows:

1. When the machine draws a lagging power factor current, the armature reaction is partly magnetizing and partly cross-magnetizing.
2. When the machine draws a leading power factor current, the armature reaction is partly demagnetizing and partly cross-magnetizing.

* Synchronous Impedance:

The actual generated voltage consists of the summation of two component voltages. One of these component voltages is the voltage that would be generated if there were no armature reaction. It is the voltage that would be generated because of only the field excitation. This component of the generated voltage is called the excitation voltage, E_{exc} .

The other component of the generated voltage is called the armature reaction voltage, E_{AR} . This is the voltage that must be added to the excitation voltage to take care of effect of

armature reaction upon the generated voltage.

$$E_a = E_{ac} + E_{AR}$$

E_{AR} is equivalent to a voltage of inductive reactance and

$$E_{AR} = -jX_{AR}I_a$$

The inductive reactance X_{AR} is a fictitious reactance which will result in a voltage in the armature circuit to account for the effect of armature reaction upon the voltage relations of the armature circuit.

In addition to the effect of armature reaction, the stator winding also has a self-inductance and a resistance.

Let L_a = Self inductance of stator winding

X_a = Self inductive reactance of stator winding.

R_a = armature (stator) resistance

The terminal voltage is given by

$$V = E_a - I_a R_a - jI_a X_a - jI_a X_{AR}$$

$I_a R_a$ = Armature resistance drop

$I_a X_a$ = Armature leakage reactance drop

$I_a X_{AR}$ = armature reaction voltage

The armature reaction effects and the leakage flux effects in the machine are both represented by inductive reactances. Therefore, it is customary to combine them into a single reactance, called the "Synchronous reactance" of the machine, X_s .

$$X_s = X_a + X_{AR}$$

$$\therefore V = E_a - jI_a X_s - I_a R_a$$

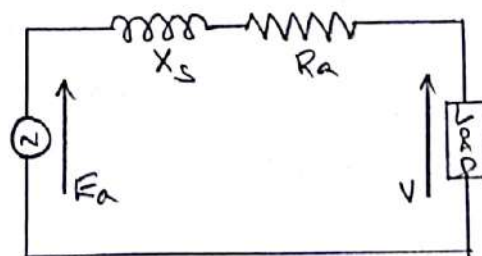
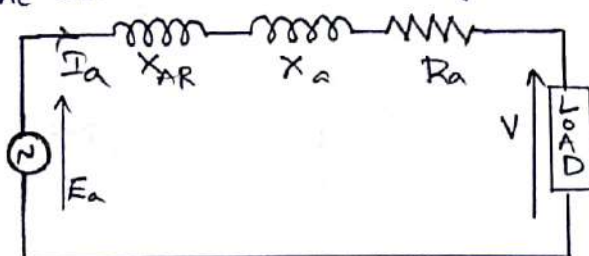
$$V = E_a - jI_a R_a - jI_a X_s = E_a - I_a (R_a + jX_s)$$

$$V = E_a - I_a Z_s \quad [\because Z_s = R_a + jX_s]$$

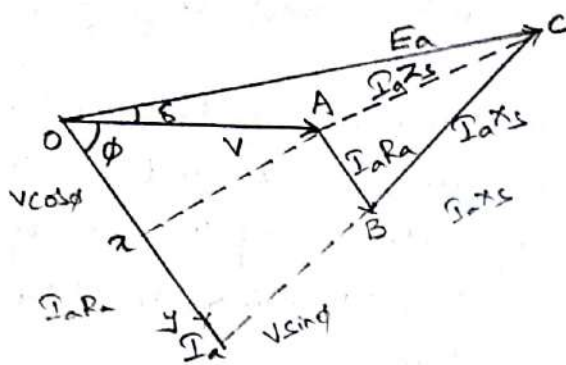
The impedance Z_s is called the "Synchronous Impedance".

* Equivalent Circuit And Phasor Diagram of A Synchronous Generator (Alternator):

The equivalent circuit of a synchronous generator is



Lagging power factor, $\cos\phi$:

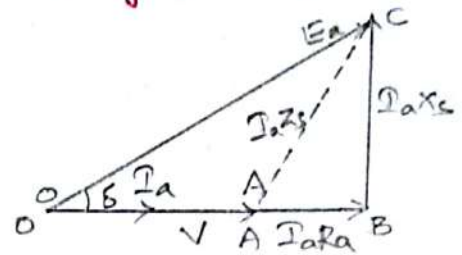


$$OC^2 = OA^2 + AC^2 = OY^2 + YC^2$$

$$E_a^2 = (V \cos\phi + I_a R_a)^2 + (V \sin\phi + I_a X_s)^2$$

$$E_a = \sqrt{(V \cos\phi + I_a R_a)^2 + (V \sin\phi + I_a X_s)^2}$$

Unity power factor, $\cos\phi = 1$:

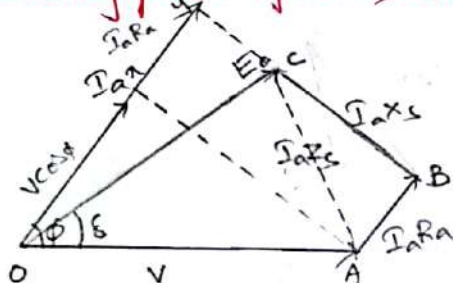


$$OC^2 = OB^2 + BC^2$$

$$E_a^2 = (V + I_a R_a)^2 + (I_a X_s)^2$$

$$E_a = \sqrt{(V + I_a R_a)^2 + (I_a X_s)^2}$$

Leading power factor, $\cos\phi$:



$$OC^2 = OY^2 + YC^2$$

$$= (OY + YZ)^2 + (YZ - CB)^2$$

$$E_a^2 = (V \cos\phi + I_a R_a)^2 + (V \sin\phi - I_a X_s)^2$$

$$E_a = \sqrt{(V \cos\phi + I_a R_a)^2 + (V \sin\phi - I_a X_s)^2}$$

* Voltage Regulation:

The Voltage regulation of a synchronous generator is the rise in the voltage at the terminals when the load is reduced from full load to zero, at constant speed and field currents.

$$\text{per cent Voltage regulation} \triangleq \frac{|E_a| - |V|}{|V|}$$

$$\text{Percentage Voltage Regulation} \triangleq \frac{|E_a| - |V|}{|V|} \times 100$$

Where $|E_a|$ = magnitude of generated voltage per phase.

$|V|$ = magnitude of rated terminal voltage per phase.

The regulation depends on the power factor of the load. For unity and lagging power factors, there is always a voltage drop with the increase of load, but for a certain leading power factor, the full-load voltage regulation is zero. In this case, the terminal voltage is the same for both full-load and no-load. At leading power factors, the voltage rises with the increase of load, and the regulation is negative.

* Determination of Voltage Regulation:

The following methods are used to determine the Voltage Regulation of Smooth cylindrical rotor type alternators:

- (a) Direct load test
- (b) Indirect methods.

(a) Direct Load Test:

The alternator is run at Synchronous Speed and its terminal Voltage is adjusted to its rated value V . The load is varied until the ammeter and wattmeter indicates the rated values at the given power factor. Then the load is removed and the speed and field excitation are kept constant. The open circuit or no-load Voltage E_a is recorded.

The Voltage regulation is found from percentage Voltage Regulation $= \frac{E_a - V}{V} \times 100$. The method of direct loading is suitable only for small alternators of power rating less than 5 kVA.

(b) Indirect Methods:

For large alternators, the three indirect methods which are used to predetermine the Voltage regulation of smooth cylindrical rotor type alternators are as follows:

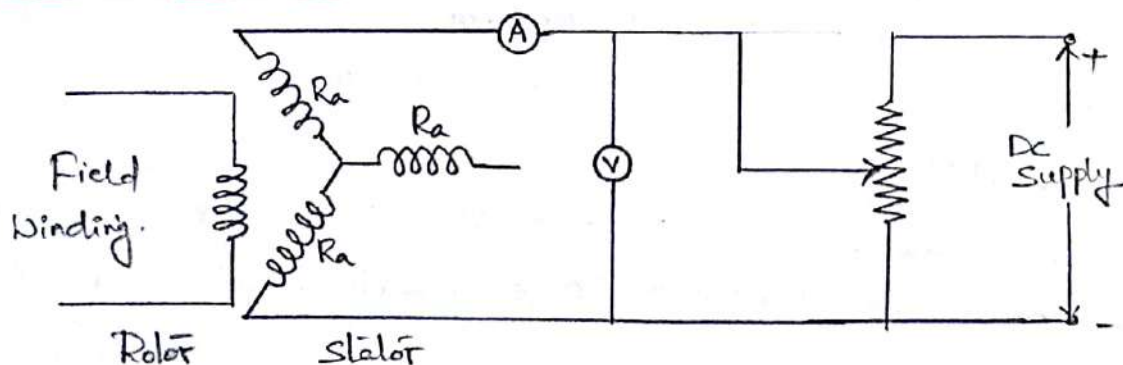
1. Synchronous Impedance Method (or) EMF method.
2. Ampere-Turn method (or) MMF Method.
3. Zero power factor method (or) Potier Method.

* Measurement of Synchronous Impedance:

The following tests are performed on an alternator to know its performance:

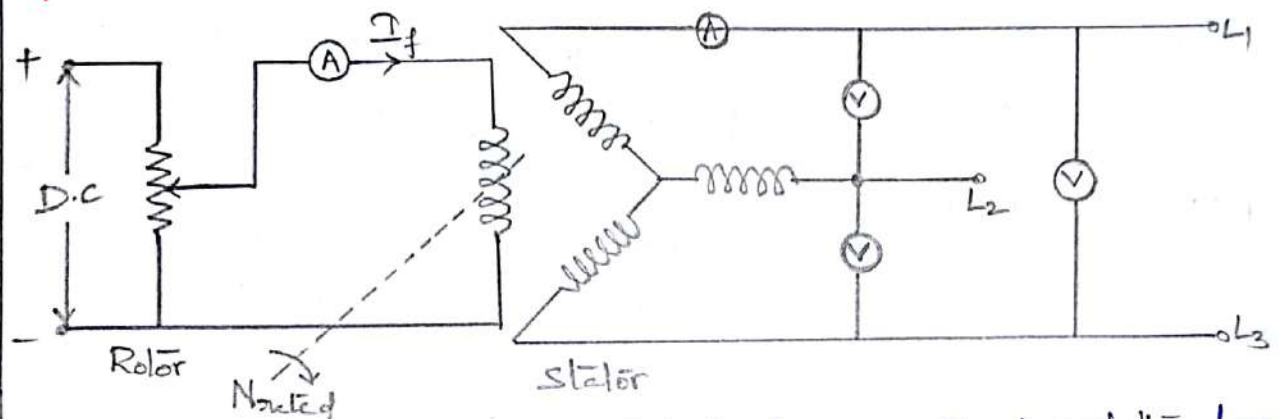
- (a) D.C. Resistance Test.
- (b) Open Circuit Test.
- (c) Short Circuit Test.

DC Resistance Test:



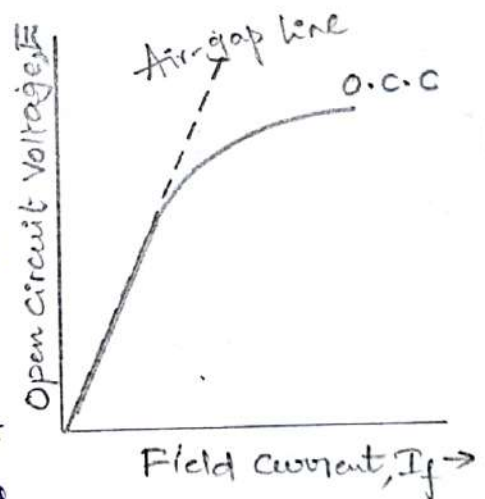
Assume that the alternator is star connected with d.c. field winding open, measure the d.c. resistance between each pair of terminals either by using ammeter-voltmeter method or by using Wheatstone's bridge. The average of three sets of resistances values R_t is taken. This value of R_t is divided by 2 to obtain the d.c. resistance per phase. The alternator should be at rest. Since the effective a.c. resistance is larger than d.c. resistance due to skin effect, therefore, the effective a.c. resistance per phase is obtained by multiplying the d.c. resistance by a factor 1.20 to 1.75 depending on the size of the machine. A typical value to use in the calculation would be 1.25.

Open Circuit test:



The alternator is run at rated synchronous speed and the load terminals are kept open. That is, all the loads are disconnected. The field current is set to zero.

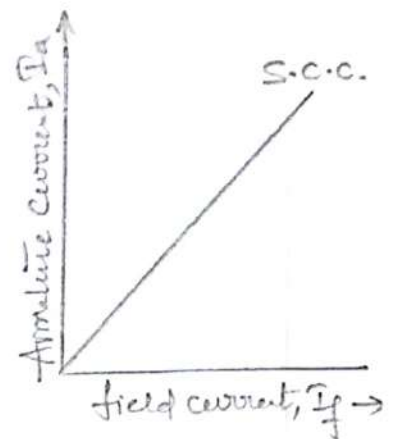
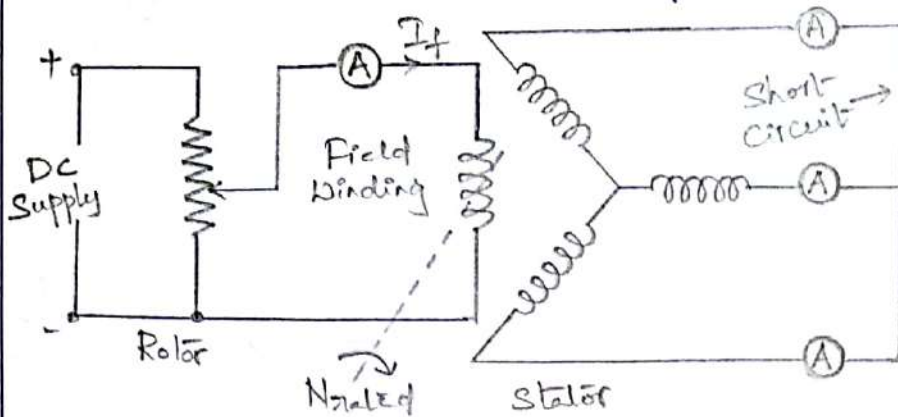
Then the field current is gradually increased in steps, and the terminal voltage E_t is measured at each step. The excitation current may be increased to get 25% more than rated voltage of the alternator. A graph is plotted between the open-circuit phase voltage $E_p (= E_t / \sqrt{3})$ and field current I_f . The characteristics curve so obtained is called open-circuit characteristics (O.C.C.).



Short Circuit Test:

The armature terminals are shorted through three ammeters. Each ammeter should have a range greater than the rated full-load value. The alternator is then run at synchronous speed. Then the field current is gradually increased in steps, and the armature -

Current is measured at each step.



The field current may be increased to get armature current upto 150% of the rated value. The field current I_f and the average of three ammeter readings at each step is taken. A graph is plotted between the armature current I_a and field current I_f . The characteristics so obtained is called short circuit characteristics.

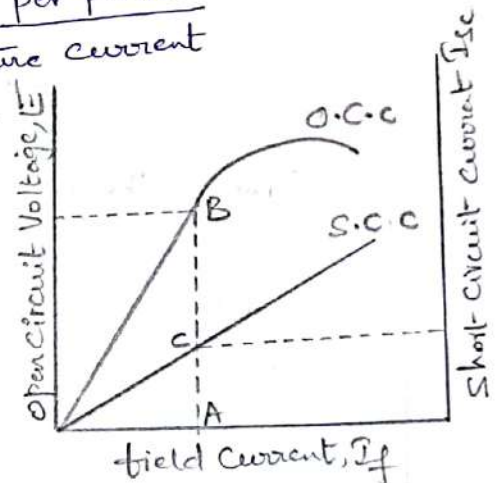
Calculation of Z_s :

$$\text{Synchronous Impedance } Z_s = \frac{\text{Open-circuit Voltage per phase}}{\text{Short Circuit Armature current}}$$

$$\text{The Synchronous Reactance, } X_s = \sqrt{Z_s^2 - R_a^2}$$

From the o.c.c & s.c.c characteristics

$$Z_s = \frac{AB \text{ (in Volts)}}{AC \text{ (in Amps)}}$$



* Synchronous Impedance Method:

Following procedural steps are

1. O.C.C is plotted from the given data
2. Similarly, S.C.C is drawn from given data. It is a straight line passing through the origin.

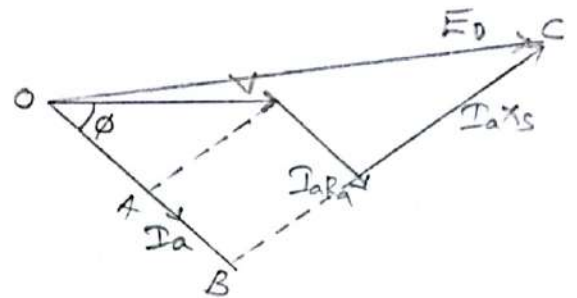
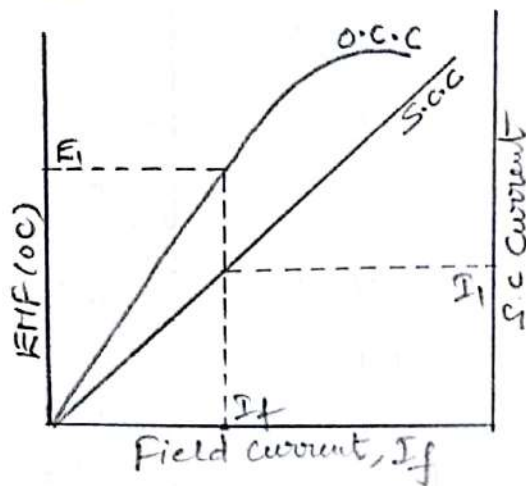
Consider a field current I_f , the O.C. Voltage corresponding to this field current is E_1 . When winding is short-circuited, the terminal voltage is zero. Hence, it may be assumed that the whole of this voltage E_1 is being used to circulate the armature short-circuit current I_1 against the synchronous impedance.

$$E_1 = I_1 Z_s$$

$$\therefore Z_s = \frac{E_1 \text{ (Open-circuit)}}{I_1 \text{ (Short-circuit)}}$$

3. From D.C. Resistance Test R_a can be found, $X_s = \sqrt{Z_s^2 - R_a^2}$

4. Knowing R_a and X_s , Vector diagram can be drawn for any load and any power factor.



From the above phasor diagram

$$OC^2 = OB^2 + BC^2$$

$$E_0^2 = (V \cos \phi + I_a R_a)^2 + (V \sin \phi + I_a X_s)^2$$

$$E_0 = \sqrt{(V \cos \phi + I_a R_a)^2 + (V \sin \phi + I_a X_s)^2}$$

$$\% \text{ Regulation} = \frac{E_0 - V}{V} \times 100.$$

Note: This method is not accurate because the value of Z_s so found is always more than its value under normal voltage conditions and saturation. Hence, the value of regulation so obtained is always more than that found from an actual test. This is why it is called pessimistic method.

Prob: Find the synchronous impedance and reactance of an alternator in which a given field current produces an armature current of 200A on short circuit and a generated emf of 50V on open circuit. The armature resistance is 0.1Ω . To what induced voltage must be alternator be excited if it is to deliver a load of 100A at a pf of 0.8 lagging, with a terminal voltage of 200V.

Sol: Given that $I_{sc} = 200A$, $I = 100A$
 $V_{oc} = 50V$
 $R_a = 0.1\Omega$

$$Z_s = \frac{V_{oc}}{I_{sc}} = \frac{50}{200} = 0.25\Omega = Z_s$$

$$X_s = \sqrt{Z_s^2 - R_a^2} = \sqrt{0.25^2 - 0.1^2} = 0.23\Omega = X_s$$

From lagging phasor diagram

$$E_0 = \sqrt{(V \cos \phi + I_a R_a)^2 + (V \sin \phi + I_a X_s)^2}$$

$$E_o = \sqrt{(2000 \times 0.8 + 100 \times 0.1)^2 + (2000 \times 0.6 + 100 \times 0.23)^2}$$

$$E_o = 222.15 \text{ V.}$$

Prob: From the following Test results, determine the voltage regulation of a 2000V, 1-phase alternator delivering a current of 100A at (i) unity p.f (ii) 0.8 leading p.f (iii) 0.71 lagging p.f

Test results: Full load current of 100A is produced on sc by a field excitation of 2.5A. An emf of 500V is produced on open circuit by the same excitation. The armature resistance is 0.8Ω

Sol: From the test results

$$Z_s = \frac{500}{100} = 5\Omega = \frac{V_{oc}}{I_{sc}} \Rightarrow Z_s = 5\Omega$$

$$X_s = \sqrt{Z_s^2 - R_a^2} = \sqrt{5^2 - 0.8^2} = 4.94\Omega = X_s$$

(i) unity p.f:

$$E_o = \sqrt{(V + I_a R_a)^2 + (I X_s)^2} = \sqrt{(2000 + 100 \times 0.8)^2 + (100 \times 4.94)^2}$$

$$= 2138 \text{ V}$$

$$\% \text{ regulation} = \frac{E_o - V}{V} \times 100 = \frac{2138 - 2000}{2000} \times 100$$

$$\% \text{ reg} = 6.9\%$$

(ii) 0.8 leading p.f:

$$E_o = \sqrt{(V \cos \phi + I R_a)^2 + (V \sin \phi - I X_s)^2}$$

$$= \sqrt{(2000 \times 0.8 + 100 \times 0.8)^2 + (2000 \times 0.6 - 100 \times 4.94)^2}$$

$$= 1822.3 \text{ V}$$

$$\% \text{ regulation} = \frac{E_o - V}{V} \times 100 = \frac{1822.3 - 2000}{2000} \times 100$$

$$\% \text{ reg} = -8.89\%$$

(iii) 0.71 lagging p.f:

$$E_o = \sqrt{(V \cos \phi + I R_a)^2 + (V \sin \phi + I X_s)^2}$$

$$E_o = \sqrt{(2000 \times 0.71 + 100 \times 0.8)^2 + (2000 \times 0.704 + 100 \times 4.94)^2}$$

$$= 2422.3 \text{ V}$$

$$\% \text{ regulation} = \frac{E_o - V}{V} \times 100 = \frac{2422.3 - 2000}{2000} \times 100$$

$$\% \text{ reg} = 21.11\%$$

Prob: A 100KVA, 3000V, 50Hz, 3-phase star connected alternator has effective armature resistance of 0.2Ω . The field current of 40A produces short circuited current of 200A and an open-ckt emf of 1040V (line voltage). Calculate the full-load voltage regulation at 0.8 pf lag and 0.8 pf lead. Draw phasor diagrams.

Prob: A 2000KVA, 11KV, 3- ϕ , star connected alternator has a resistance of 0.3Ω & reactance of 5Ω per phase. It delivers full-load current at 0.8 lag power factor at rated voltage. Compute the terminal voltage for the same excitation and load current at 0.8 pf leading.

Sol: At 0.8 pf lagging:

$$I = \frac{2000 \times 10^3}{\sqrt{3} \times 11 \times 10^3} = 105A$$

$$\text{Terminal voltage} = 11K/\sqrt{3} = 6350V.$$

$$\therefore E_0 = \sqrt{(V \cos \phi + I_a R_a)^2 + (V \sin \phi + I_a X_s)^2}$$

$$E_0 = \sqrt{(6350 \times 0.8 + 105 \times 0.3)^2 + (6350 \times 0.6 + 105 \times 5)^2}$$

$$E_0 = 6700V.$$

At 0.8 pf leading:

$$E_0 = \sqrt{(V \cos \phi + I_a R_a)^2 + (V \sin \phi - I_a X_s)^2}$$

$$6700 = \sqrt{(V \times 0.8 + 105 \times 0.3)^2 + (V \times 0.6 - 105 \times 5)^2}$$

$$\boxed{V = 6975V}$$

Prob: In a single phase alternator, a given field current produces an armature current of 250A on short circuit and a generated emf of 1500V on open-circuit. Calculate the terminal p.d when a load of 250A at 6.6KV and 0.8 pf lagging is switched off. Calculate also the regulation of the alternator at the given load.

* M.M.F Method (or) Ampere-Turn Method:

This method also utilizes O.C & S.C data, but is the converse of the EMF Method in the sense that armature leakage reactance is treated as an additional armature reaction.

Now, field A.T. required to produce a voltage of V on full load is the vector sum of the following:

- (i) Field A.T. required to produce a voltage of V on no load. This can be found from O.C.C. and
- (ii) Field A.T. required to overcome the demagnetising effect of armature reaction on full-load. This value is found from S.C. Test. The field A.T. required to produce full load current on short-circuit balances the armature reaction and the "Impedance Drop".

The impedance drop can be neglected because R_a is usually very small and X_s is small under short-circuit conditions. Hence, p.f. on short circuit is almost zero lagging and the field A.T. are used entirely to overcome the armature reaction which is wholly demagnetising. In other words, the demagnetising armature A.T. on full load are equal and opposite to the field A.T. required to produce full-load current on short-circuit.

Prob: A 3.5 MVA, Y-connected alternator rated at 4160 V at 50 Hz has the open circuit characteristics given by the following data:

Field current (A)	50	100	150	200	250	300	350	400	450
E.M.F (Volt)	1620	3150	4160	4750	5130	5370	5550	5650	5750

A field current of 200 A is found necessary to circulate full load current on short circuit of the alternator. Calculate by (i) synchronous Impedance Method & (2) ampere-Turn method the full load voltage regulation at 0.8 pf lagging. Neglect resistance. Comment on the result obtained.

Sol: Given that Rating = 3.5 MVA
Rated Voltage = 4160 V, 50 Hz

Field current (A)	50	100	150	200	250	300	350	400	450
EMF (V) in ph	935.3	1818.7	2401.8	2742.5	2962	3100	3204	3262	3320

field current to circulate full load on S/C = 200 A = I_f

$$\text{Short Circuit Current} = \frac{3.5 \text{ M}}{\sqrt{3} \times 4160} = 485.76 \text{ A}$$

$$E_o = \sqrt{(V \cos \phi + I_a R_a)^2 + (V \sin \phi + I_a X_s)^2}$$

$$= \sqrt{(2402 \times 0.8 + 0)^2 + (2402 \times 0.6 + 485.76 \times 5.65)^2}$$

$$= 4605.7 \text{ V.}$$

$$Z_s = \frac{V_{oc}}{I_{sc}} = \frac{2742.5}{485.76} = 5.65 \Omega$$

$$R_a = 0, X_s = Z_s = 5.65 \Omega$$

I_{f1} = field current for open circuit voltage = 150 A.

$$I_f = \sqrt{I_{f1}^2 + I_{f2}^2 + 2 I_{f1} I_{f2} \cos(180 - (90 + \phi))} = \sqrt{150^2 + 200^2 + 2 \times 150 \times 200 \times \cos(180 - 126.52)}$$

$$= 313.84 \text{ A}$$

From the graph, at $I_f = 313.84$, $E_o = 3160 \text{ V.}$

$$\% \text{ Reg} = \frac{3160 - 2402}{2402} \times 100 = 31.5\% = \% \text{ Reg}$$

Prob: The open circuit and short circuit test readings for a 3- ϕ star connected, 1000 KVA, 2000 V, 50 Hz, Synchronous generator are:

Field Amps:	10	20	25	30	40	50
O.C Terminal voltage:	800	1500	1760	2000	2350	2600
S.C. Arm current:	-	200	250	300	-	-

The armature effective resistance is 0.2 Ω per phase. Draw the characteristics curve and estimate the full load percentage regulation at (a) 0.8 pf lagging (b) 0.8 p.f leading.

Sol: Given that.

$$\text{Short Circuit full load current, } = \frac{1000 \text{ K}}{\sqrt{3} \times 2000} = 288.68 \text{ A}$$

Field Amps :	10	20	25	30	40	50
O.C. Terminal voltage:	461.9	866	1016.2	1154.7	1356.8	1506.15
S.C. Current:	-	200	250	300	-	-

$$\text{O.C. Voltage} = \text{rated voltage} + \text{Drop} = 1154.7 + 288.68 \times 0.2 \times 0.8 = 1201 \text{ V.}$$

I_{f1} = field current of O.C Voltage = 30 A

I_{f2} = field current of S.C current = 28.5 A (from graph)

(a) for 0.8 pf lagging:

$$\phi = 36.86^\circ \text{ lag}$$

$$I_f = \sqrt{I_{f1}^2 + I_{f2}^2 + 2 I_{f1} I_{f2} \cos(180 - (90 + \phi))}$$

$$= \sqrt{30^2 + 28.5^2 + 2 \times 30 \times 28.5 \times \cos(180 - 126.86^\circ)} = 54.32 \text{ A}$$

$$I_f = 54.32 \text{ A}$$

At $I_f = 54.32 \text{ A}$, $E_o = 1550 \text{ V}$

$$\% \text{ Reg} = \frac{E_o - V}{V} \times 100 = \frac{1550 - 1154.73}{1154.73} \times 100 = 34.23\% = \% \text{ Reg}$$

(b) 0.8 pf leading: $\phi = 36.86^\circ$ lead

$$I_f = \sqrt{I_{f1}^2 + I_{f2}^2 + 2I_{f1}I_{f2}\cos(180 - (90 - \phi))}$$

$$= \sqrt{32^2 + 28.5^2 + 2 \times 32 \times 28.5 \times \cos(180 - 53.14)} = 27.24 \text{ A}$$

at $I_f = 27.24 \text{ A}$, the $E_o = 1080 \text{ V}$

$$\% \text{ Reg} = \frac{E_o - V}{V} \times 100 = \frac{1080 - 1154.7}{1154.7} \times 100$$

$$\% \text{ Reg} = -6.4\%$$

Prob: A 30KVA, 440V, 50Hz, Y-Connected synchronous generator gave the following test data:

Field Current (A):	2	4	6	7	8	10	12	14
Terminal Volt (V):	155	287	395	440	475	530	570	592
S.C. Current :	11	22	34	40	46	57	69	80

Resistance b/w the any two terminals is 0.3Ω .

Find the regulation at full load 0.8 pf lag by (a) Synchronous Impedance Method & (b) Ampere turn Method. Take Z_s corresponding to S.C. current of 80A.

Sol: Full load current, $I = \frac{30\text{K}}{\sqrt{3} \times 440} = 39.36 \text{ A}$

Amp-Turn Method:

Field Curr (A):	2	4	6	7	8	10	12	14
Terminal Volt (V):	89.49	165.7	228	254	274.2	306	329	342
S.C current (A):	11	22	34	40	46	57	69	80

$$Z_s = \frac{V_{oc}}{I_{sc}} = \frac{254}{80} = 3.175$$

$$X_s = \sqrt{3.175^2 - 0.3^2} = 3.162$$

O.C. Voltage = rated Voltage + Drop.

$$= 254.04 + 39.36 \times 0.3 \times 0.8 = 263.5 \text{ V}$$

I_{f1} = field current of O.C Voltage = 7.75A

I_{f2} = field current of S.C current = 6.95A

$\phi = 36.86^\circ$ lag

$$I_f = \sqrt{7.75^2 + 6.95^2 + 2 \times 7.75 \times 6.95 \times \cos(180 - (90 + \phi))} = 13.15 \text{ A}$$

At $I_f = 13.15 \text{ A}$, the EMF = $E_o = 338 \text{ V}$

$$\% \text{ Reg} = \frac{E_o - V}{V} \times 100 = \frac{338 - 254}{254} \times 100 = 33.07\% = \% \text{ Reg}$$

Synchronous Impedance Method:

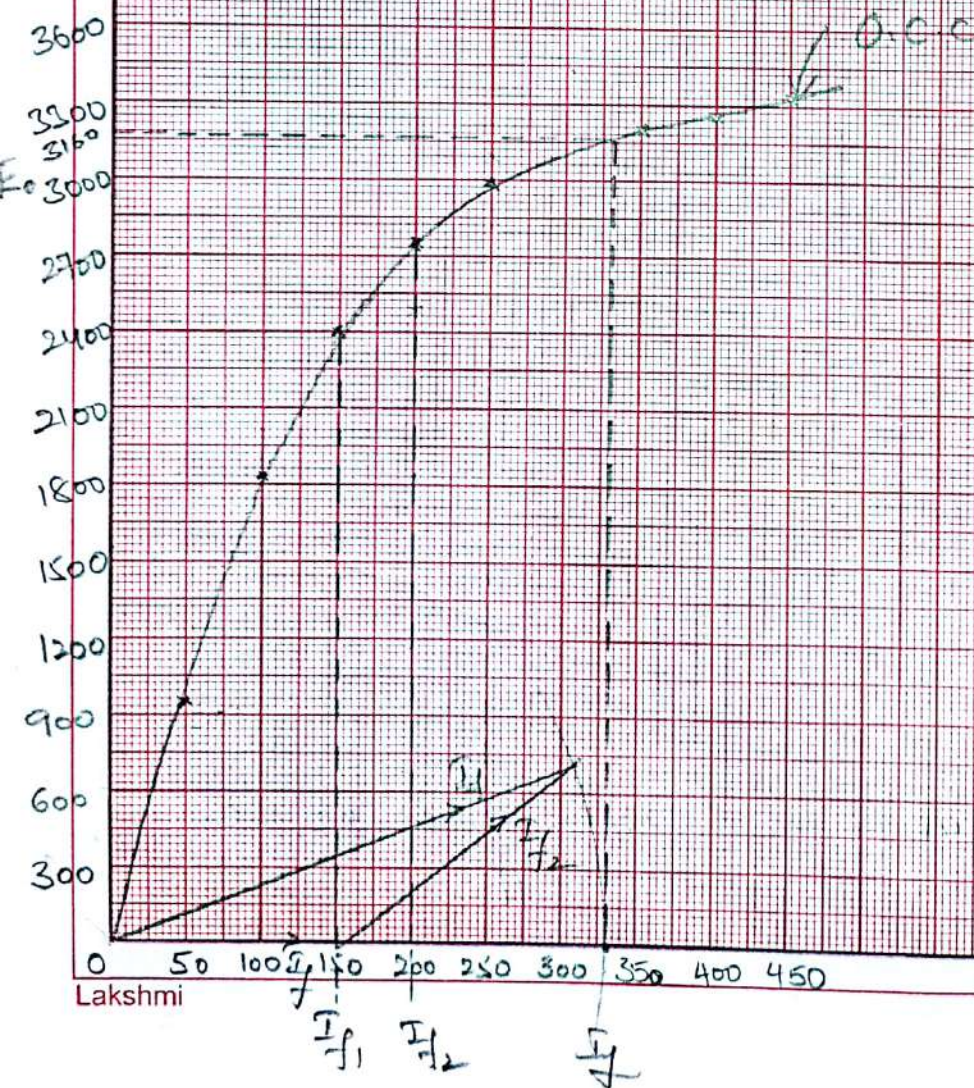
$$Z_s = \frac{V_{oc}}{I_{sc}} = \frac{254}{80} = 3.175, R_a = 0.3\Omega, X_s = \sqrt{3.175^2 - 0.3^2} = 3.162$$

$$E_o = \sqrt{(254 \times 0.8 + 39.36 \times 0.3)^2 + (254 \times 0.6 + 39.36 \times 3.16)^2} = 350.47 \text{ V}$$

$$\% \text{ Reg} = \frac{E_o - V}{V} \times 100 = \frac{350.47 - 254}{254} \times 100 = 37.98\% = \% \text{ Reg}$$

% Reg with MMF Method

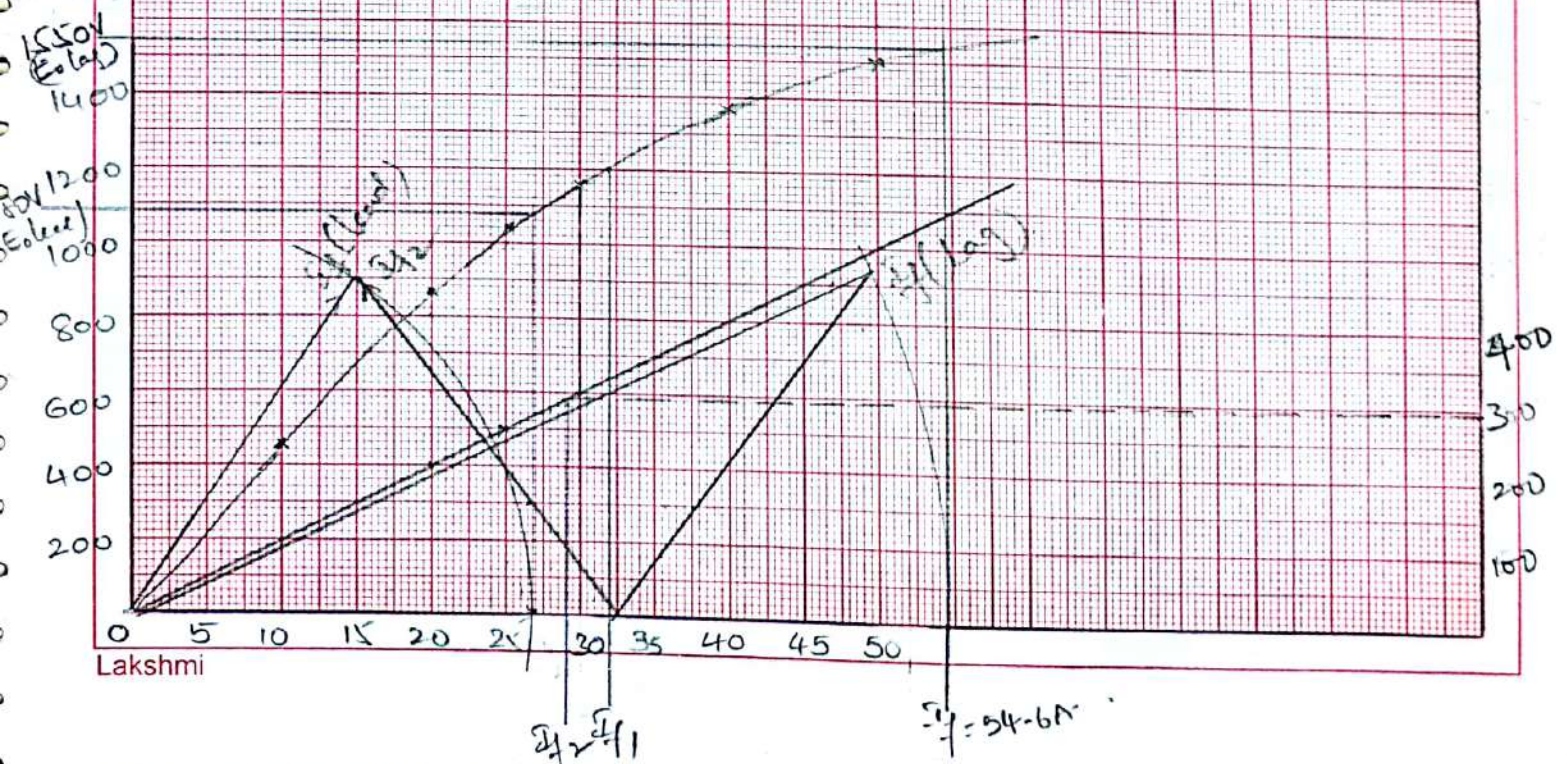
①



% Reg With MMF Method

(2)

$T_1 = 30^\circ$
 $T_2 = 28^\circ$



Prob: Find the regulation by the ZPF method of 5000KVA, 6600V, 3- ϕ star connected alternator at full load, unity power factor having the following test data:

Field current (Amp):	32	50	75	100	140
O.c terminal voltage (V):	3100	4900	6600	7500	8300
F.L current ZPF tests: line pt in (V)	0	1850	4250	5800	7000

Neglect the armature resistance:

And also find regulation at 0.8 pf lagging.

Sol:

Field current (A)	O.c terminal voltage (V) (line)	O.c terminal voltage (V) (phase)	ZPF Voltage (V) (line)	ZPF Voltage (V) (phase)
32	3100	1789.8	0	0
50	4900	2829	1850	1069
75	6600	3810.6	4250	2453.8
100	7500	4330.12	5800	3348.6
140	8300	4792	7000	4041.5

From the O.C.C and ZPF graph.

Armature reactance drop, $DC = 1.3 \text{ cm} = 520 \text{ V}$.

Armature reactance current, $BD = 2.3 \text{ cm} = 23 \text{ A}$

From the graph, air gap voltage, $E_{ap} = V_{ph} + j(I_a X_L) + I_a R_a$
 $= V_{ph} + I_a Z_L$ [$R_a \rightarrow$ Neglected].

For unity power factor:

$$V_{ph} = V_{ph} \angle 0^\circ = 3810.6 \angle 0^\circ \text{ V}$$

$$E_{ap} = 3810 \angle 0^\circ + j520 = 3846 \angle 7.77^\circ \text{ V}$$

Corresponding E_{ap} , the field current, $I_{f1} = 79 \text{ A} = 79 \angle 7.77^\circ \text{ A}$

Corresponding Arm reactance field current, $I_{f2} = 23 \text{ A} = 23 \angle 0^\circ \text{ A}$.

$$\text{Field current of Alternator at } E_o, I_f = I_{f1} - I_{f2} \\ = 79 \angle 7.77^\circ - 23 \angle 0^\circ = 85.2 \angle 3.3^\circ \text{ A}$$

Corresponding I_f , voltage, $E_o = 4080 \text{ V}$

$$\% \text{ Regulation} = \frac{E_o - V}{V} \times 100 = \frac{4080 - 3810.6}{3810.6} \times 100$$

$$\% \text{ Reg} = 7.06\%$$

For 0.8 lag power factor:

$$V_{ph} = V_{ph} \angle +36.86^\circ V$$

$$E_{gp} = V_{ph} + jI_a X_L = 3810.6 \angle +36.86^\circ + j520 = 4143.5 \angle 42.6^\circ V$$

$$\text{Correspond } E_{gp}, I_{f1} = 89 A = 89 \angle 90^\circ + 42.6^\circ = 89 \angle 132.6^\circ A$$

$$\therefore \text{field current for } E_o, I_f = I_{f1} - I_{f2} = 89 \angle 132.6^\circ - 23 \angle 0^\circ = 105.93 \angle 142^\circ A$$

$$\text{Correspond } I_f, E_o = 4400 V.$$

$$\% \text{ Reg} = \frac{E_o - V}{V} \times 100 = \frac{4400 - 3810.6}{3810.6} = 15.46\%$$

$$\% \text{ Reg} = 15.46\%$$

For 0.8 lead power factor:

$$V_{ph} = V_{ph} \angle -36.86^\circ V$$

$$E_{gp} = V_{ph} + jI_a X_L = 3810.6 \angle -36.86^\circ + j520 = 3523.3 \angle -30^\circ V$$

$$\text{Corresponding } E_{gp}, I_{f1} = 68 A = 68 \angle 90^\circ - 30^\circ = 68 \angle 60^\circ A$$

$$\text{field current for } E_o, I_f = I_{f1} - I_{f2} = 68 \angle 60^\circ - 23 \angle 0^\circ = 59.91 \angle 79.42^\circ A$$

$$\text{Corresponding } I_f, E_o = 3280 V.$$

$$\% \text{ Reg} = \frac{E_o - V}{V} \times 100 = \frac{3280 - 3810.6}{3810.6} \times 100$$

$$\% \text{ Reg} = -13.92\%$$

prob: A 1000 KVA, 11000 V, 3- ϕ , 50 Hz star connected turbo-generator has an effective resistance of 2 Ω /ph. The o.c.c. & Zpu pf full load data is as follows:

O.C. Volt (V)	5805	7000	12,550	13,755	15,000
Field current (A)	40	50	110	140	180
F.L Zpu p.f (V)	0	1500	8500	10,500	12,400.

Estimate the % regulation for F.L. at 0.8 pf lagging.

Field current (A)	O.C. Voltage (V) (line)	O.C. Voltage (V) (ph)	Zpu Voltage (V) (line)	Zpu Voltage (V) (ph)
40	5805	3351.5	0	0
50	7000	4041.5	1500	866
110	12550	7245.7	8500	4907.5
140	13755	7941.5	10500	6062.2
180	15000	8660.3	12400	7159.1

Voltage, $V = 11000V$, Phase Voltage, $V_{ph} = 6350.85V$

From the graph

Armature Reaction drop, $D_c = 2.3cm = 1150V$.

Armature Reaction current, $I_{f2} = BD = 1.35cm = 27A$

Armature Resistance drop, $I_a R_a$, $BD = 1.35cm = 675V$.

Air gap Voltage, $E_{gp} = V_{ph} + I_a R_a \text{ drop} + \text{Arm Reactance drop at } 0.8 \text{ pf lag}$

$$E_{gp} = 6350.85 \angle 36.86^\circ + (675 + j1150)$$

$$E_{gp} = 7598.2 \angle 40.75^\circ V$$

From the graph, at E_{gp} , the field current, $I_{f1} = 124 \angle (90^\circ + 40.75^\circ)$

$$I_{f1} = 124 \angle 130.75^\circ A$$

$\therefore$ field current of E_o , $I_f = I_{f1} - I_{f2}$

$$= 124 \angle 130.75^\circ - 27 \angle 0^\circ = 143 \angle 138.97^\circ A$$

At field current, $I_f = 143 \angle 138.97^\circ A$, $E_o = 8000V$ [from o.c.c. Graph].

$$\% \text{ Reg} = \frac{E_o - V}{V} \times 100 = \frac{8000 - 6350.85}{6350.85} \times 100$$

$$\boxed{\% \text{ Reg} = 25.96\%}$$

Prob:

(3)

A 220V, 50Hz, 6-pole star connected alternator with Ohmic resistance of $0.06\Omega/\text{ph}$, gave the following data for open circuit, s/c & full load Zero power factor characteristics:

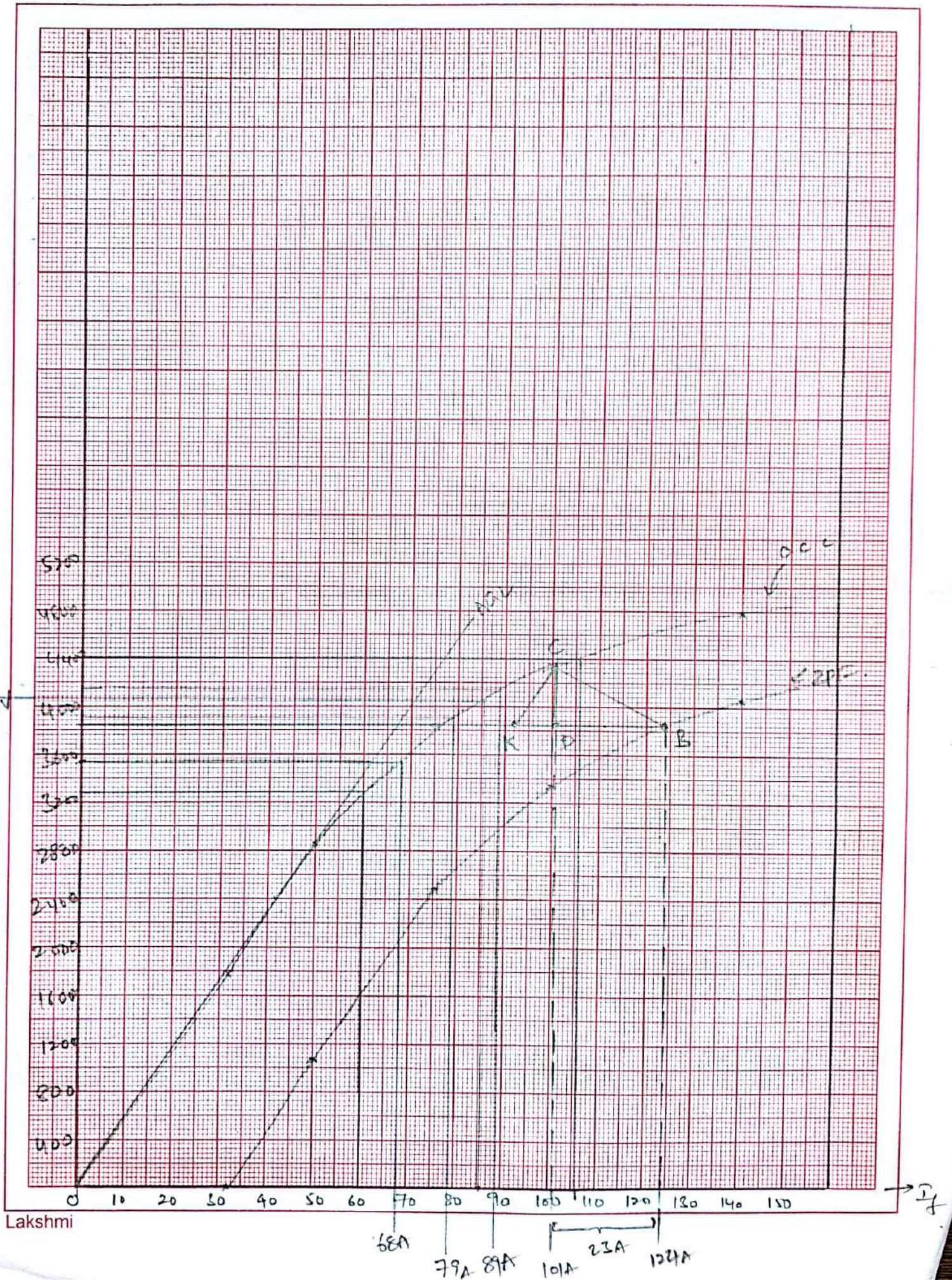
Field Current (A)	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.8	2.2	2.6	3.0	3.4
Open Ckt, E_f Voltage (V)	29 16.74	58 33.5	87 50.2	116 66.9	146 84.3	172 99.3	194 112	232 133.9	261.5 150.99	284 164.3	300 173.2	310 178.98
Short Ckt (I_{sc}) Current (A)	6.6	13.2	20	26.5	32.4	40	46.3	59	-	-	-	-
Zpf Terminal Voltage in (V)	-	-	-	-	-	0 0	29 16.74	88 50.8	140 80.8	177 102.2	208 120.1	230 122.8

Find the % Voltage regulation at full load current of 40A at Power factor of 0.8 lag by (1) emf method (2) MMF method (3) Zpf method

Ans [53.9%, 28.75%, 33.1%]

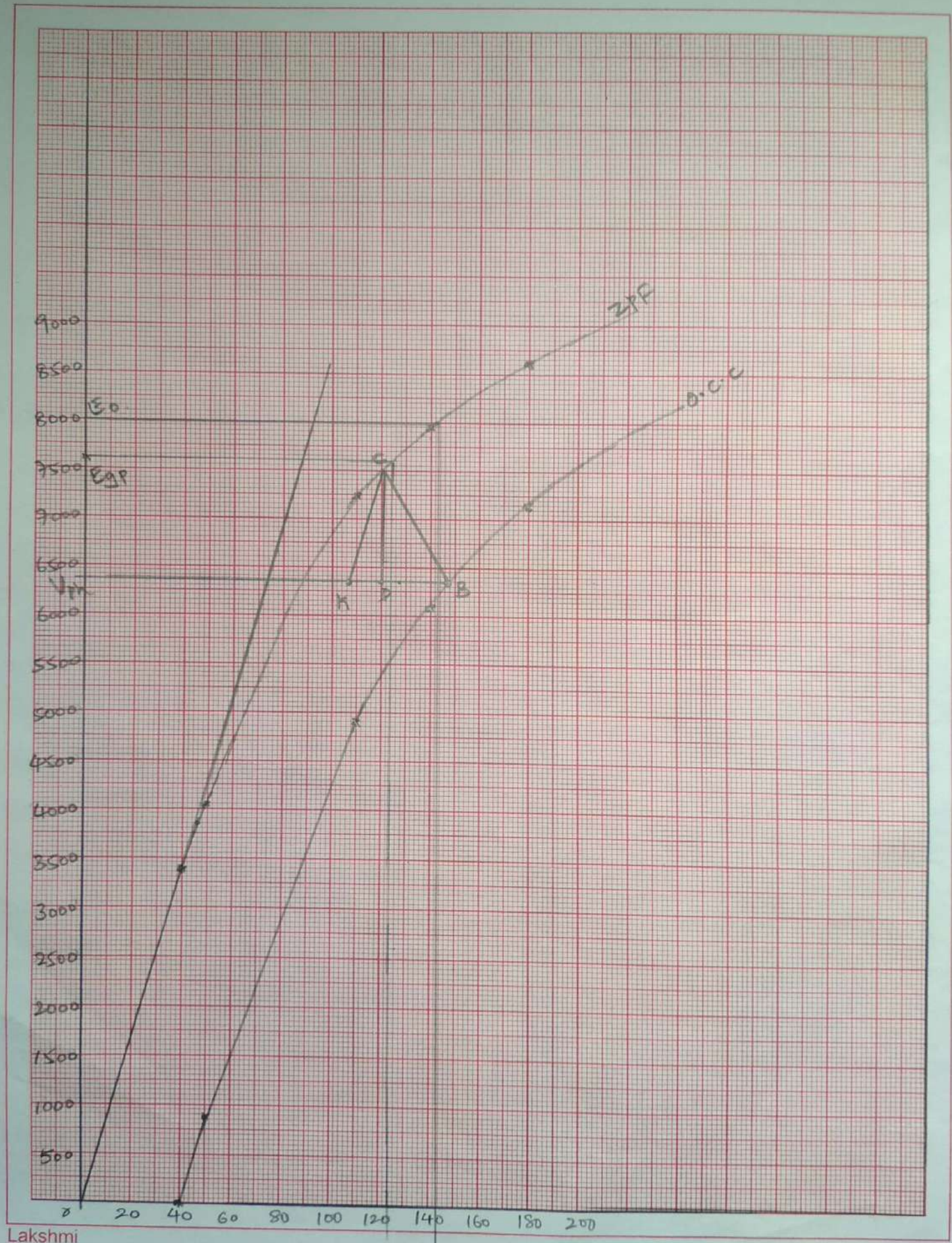
①

% Reg with ZPF method



21

ZPP Method



Lakshmi

24A NBA

Sol-

Voltage, $V_{ph} = 127V$, full load current, $I_a = 40A$, $R_a = 0.06\Omega$
 from the graph Armature reactance drop = $2.75cm = 2.75 \times 10 = 27.5V$
 Armature Resistance drop = $I_a R_a = 40 \times 0.06 = 2.4V$

Air gap induced emf, $E_{gp} = V_{ph} + I_a R_a + j I_a X_s$

at 0.8 pf lag: $E_{gp} = 127 \angle 36.86^\circ + 2.4 + j 27.5$
 $= 146.86 \angle 44.9^\circ V$

E_{gp} field current, $I_{f1} = 2.08 \angle (60^\circ + 44.9^\circ) = 2.08 \angle 134.9^\circ A$

Drop field current, $I_{f2} = 4.35cm = 4.35 \times 0.2 = 0.87 \angle 0^\circ A$

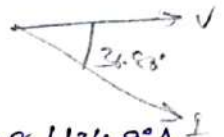
No load Induced emf current, $I_f = I_{f1} - I_{f2}$
 $= 2.08 \angle 134.9^\circ - 0.87 \angle 0^\circ$

$I_f = 2.76 \angle 147.78^\circ A$

Corresponding I_f , No load Induced emf, $E_0 = 168V$

% Regulation = $\frac{E_0 - V}{V} \times 100$
 $= \frac{168 - 127}{127} \times 100$

% Reg = 32.28%

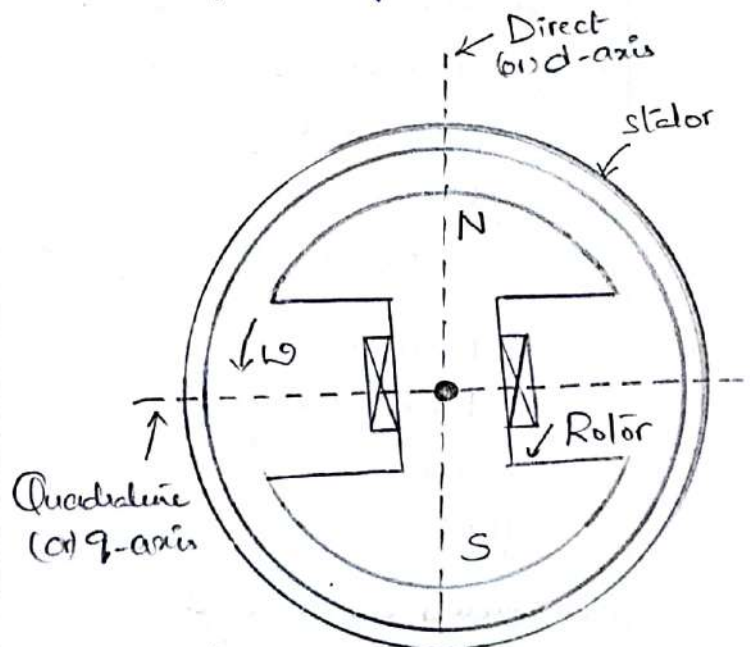


* Two-Reactance Concept for Salient pole Synchronous Machines:

* In case of a multi-pole machine having cylindrical rotor, the air gap is uniform and as a consequence its reactance remains the same, irrespective of the position occupied by the rotor. Thus the effect of armature reaction, fluxes and voltages induced can be treated in a single way with the concept of a synchronous reactance and taking it as constant for all positions of field poles with the armature.

* In case of Salient pole machine, the reluctance of the magnetic path on which the emf acts is different along the "direct-axis" and "quadrature axis".

- The reluctance of the direct-axis magnetic circuit is due to yoke and teeth



Of the stator, air gap pole and Core of the rotor.
- The reluctance in quadrature axis is mainly due to large air gap in inter polar space.

* When there is a phase difference between armature and excitation voltage (some angle b/w 0° & 90°), the armature mmf will have a direct acting as well as a quadrature component.

- The "Direct acting Component" is proportional to the Sine of the phase angle between the armature current and excitation voltage.

- The "Quadrature (or cross) Component" is proportional to the Cosine of the angle, between the armature current and the excitation voltage.

* The two-reactance concept for salient pole machines replaces the effect of armature reaction by two fictitious voltage; these reactance voltages are respectively, $I_d X_{ad}$ and $I_q X_{aq}$, where I_d and I_q are components of the armature current along direct and quadrature axes respectively.

The Synchronous reactance for each component of the armature mmf is as follows:

Direct axis synchronous reactance, $X_d = X_{ad} + X_L$

Quadrature axis synchronous reactance, $X_q = X_{aq} + X_L$

Where X_L = Armature leakage reactance.

The values of X_d & X_q are determined by applying a balanced reduced external voltage, to an unexcited machine at a speed a little less than synchronous speed, (the slip being less than 1%). Applied voltage to the armature, armature current and the voltage induced in the field winding are measured by the oscillographs. From oscillograph records:

$$X_d = \frac{\text{Maximum Voltage}}{\text{Minimum Current}} \quad ; \quad X_q = \frac{\text{Minimum Voltage}}{\text{Maximum Current}}$$

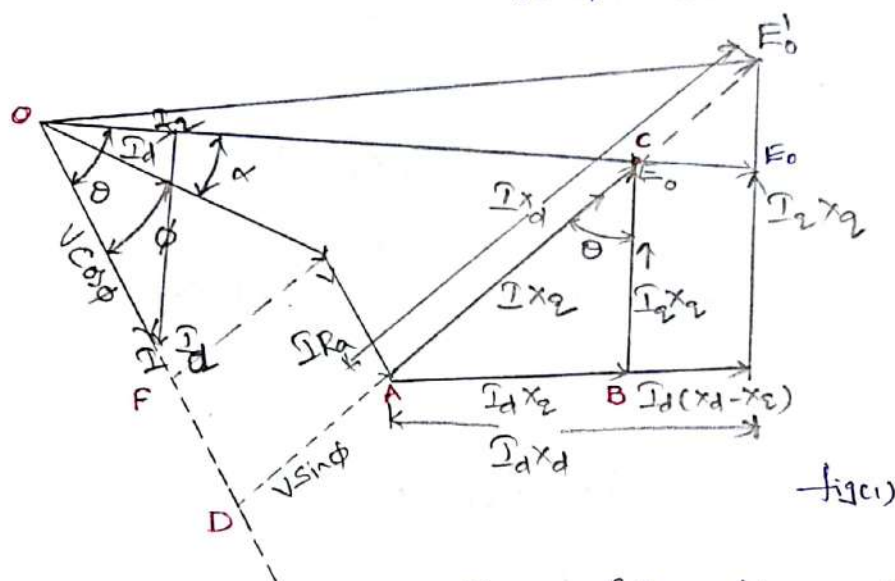
Construction of Two-reactance Diagram from Test Data:

After finding the values of X_d & X_q the two reactance may be constructed as follows:

From the phasor diagram,

$$\Delta ODC, \tan \theta = \frac{CD}{OD} = \frac{CA+AD}{OF+FD} = \frac{V \sin \phi + I X_q}{V \cos \phi + I R_a} \quad (\text{Generating})$$

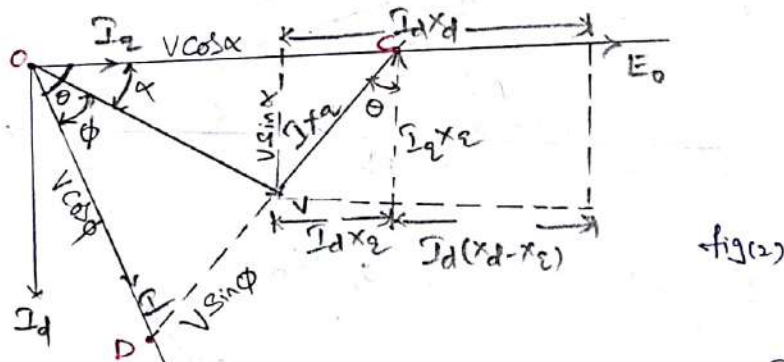
$$= \frac{V \sin \phi - I X_q}{V \cos \phi - I R_a} \quad (\text{Motoring}).$$



and $I_d = I \sin \theta$, $I_q = I \cos \theta$ (from phasor diagram).

If armature resistance is neglected. The phasor diagram is:

In fig(1) the extremity of E_0' is shown on the line perpendicular to $I_d X_d$ and it is slightly greater than E_0 , making



The regulation obtained by the Synchronous Impedance method greater than the than that of obtained by two reaction method. The difference depends on the saliency (ratio of pole pitch to pole arc) and increase with the increase in the value of saliency. The major difference b/w two methods is the magnitude of the torque angle, which is much larger when determined from the synchronous impedance phasor than when found from the two reaction diagram.

* The angle θ b/w E_0 & I called the "internal power factor angle" while α , the angle b/w E_0 & V is called the "load or power angle".

from fig (2) (Phasor Diagram without R_a).

$$E_o = V \cos \alpha \pm I_d X_d$$

$$\Rightarrow V \cos \alpha + I_d X_d \rightarrow \text{Generator mode}$$

$$\Rightarrow V \cos \alpha - I_d X_d \rightarrow \text{Motor mode}$$

from fig (1) [with R_a]

$$E_o = V \cos \alpha \pm I R_a \cos \theta \pm I_d X_d$$

$$E_o = \begin{cases} V \cos \alpha + I R_a \cos \theta + I_d X_d \rightarrow \text{Generator} \\ V \cos \alpha - I R_a \cos \theta - I_d X_d \rightarrow \text{Motor} \end{cases}$$

$I R_a \quad I \cos \theta = I_a$

Power developed by a Synchronous Generator:

Power developed per / ph $P_{\text{phase}} = V I \cos \phi$

from fig (2). $I_q X_q = V \sin \alpha$

$$I_d X_d = E_o - V \cos \alpha$$

$$I \cos \phi = I_d \sin \alpha + I_q \cos \alpha$$

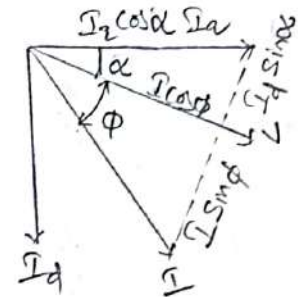
$$V I \cos \phi = V I_d \sin \alpha + V I_q \cos \alpha$$

$$P_{\text{ph}} = V \left[\frac{E_o - V \cos \alpha}{X_d} \right] \sin \alpha + V \left[\frac{V \sin \alpha}{X_q} \right] \cos \alpha$$

$$P_{\text{ph}} = \frac{V E_o}{X_d} - \frac{V^2}{X_d} \sin \alpha \cos \alpha + \frac{V^2}{X_q} \sin \alpha \cos \alpha$$

$$P_{\text{ph}} = \frac{V E_o}{X_d} + V^2 \sin \alpha \cos \alpha \left[\frac{1}{X_q} - \frac{1}{X_d} \right]$$

$$P_{\text{ph}} = \frac{V E_o}{X_d} + \frac{V^2}{2} \left[\frac{1}{X_q} - \frac{1}{X_d} \right] \sin 2\alpha$$



for 3- ϕ

$$P = \frac{3 V E_o}{X_d} + \frac{3 V^2}{2} \left[\frac{1}{X_q} - \frac{1}{X_d} \right] \sin 2\alpha$$

From the above expression.

(1) First term represents power due to field excitation.

(2) Second term gives the reluctance power (i.e., power due to saliency).

- In case of a cylindrical motor machine $X_d = X_q$, and so the 2nd term becomes zero and the power is given by the 1st term only.
- In the absence of field excitation (i.e. E_o) the 1st term becomes zero and the power is given by the second term.

* SCR of the Alternator (Short Circuit Ratio):

SCR of a Synchronous machine is defined as the ratio of field current to produce rated voltage on open circuit to field current required to circulate rated current on short-circuit while the machine is mechanically driven at synchronous speed.

$$SCR = \frac{1}{\left[\frac{\text{Per unit Voltage on open circuit}}{\text{Corresponding per unit current on S/C}} \right]} = \frac{1}{Z_s} = \frac{1}{X_s} (R_a \ll X_s).$$

Prob: A 3- ϕ generator rated at 25 MVA, 0.8 power factor lag, 13.8 kV is operating at normal voltage and rated load. The direct axis sync. reactance is 7.62Ω , quadrature axis sync. reactance is 4.57Ω and armature resistance is 0.15Ω per phase. Determine the direct axis and quadrature axis components of armature current and internal induced voltage. Also find the regulation. [EM-III, May-2017, Supply].

Sol: Given data

3- ϕ
 $S = 25 \text{ MVA}$
 Load power factor, $\cos \phi = 0.8 \Rightarrow \phi = 36.86^\circ$
 Voltage, $V = 13.8 \text{ kV (line)}$
 $V_{ph} = 7967.67 \text{ V}$
 $X_d = 7.62 \Omega$, $X_q = 4.57 \Omega$, $R_a = 0.15 \Omega$

From the phasor diagram

$$\theta = \tan^{-1} \left[\frac{V \sin \phi + I X_q}{V \cos \phi + I R_a} \right]$$

$$I = \frac{25 \text{ M}}{\sqrt{3} \times 13.8 \text{ k}} = 1045.95 \text{ A}$$

$$\therefore \theta = \tan^{-1} \left[\frac{7967.67 \times 0.6 + 1045.95 \times 4.57}{7967.67 \times 0.8 + 1045.95 \times 0.15} \right] = 55.66^\circ$$

Input power factor = $\cos \theta = 0.564$ lag.

$$\therefore \text{load angle, } \alpha = \theta - \phi \\ = 55.66^\circ - 36.86^\circ = 18.8^\circ$$

$$\text{Induced EMF, } E_o = V \cos \alpha + I R_a \cos \theta + I_d X_d$$

$$\text{Direct axis current, } I_d = I \sin \theta = 1045.95 \times \sin(55.66^\circ) = 863.65 \text{ A}$$

$$\text{Quadrature axis current, } I_q = I \cos \theta = 1045.95 \times \cos(55.66^\circ) = 590.02 \text{ A}$$

$$E_o = 7967.67 \times \cos 18.8^\circ + 1045.95 \times 0.15 \times 0.564 + 863.65 \times 7.62$$

$$E_o = 14212.08 \text{ V (ph)}$$

$$E_o = 24615.34 \text{ V (line)}$$

$$\% \text{Reg} = \frac{E_o - V}{V} \times 100 = \frac{14212 - 7967.67}{7967.67} \times 100 = 78.37\%$$

$$\% \text{Reg} = 78.37\%$$

Prob: A 3-phase, star connected Salient Pole Sync. generator is driven at a speed near synchronous with the field current circuit open and the stator is supplied from a balanced 3- ϕ supply. Voltmeter connected across the line gave minimum and maximum readings of 1196V and 1217V. The line current fluctuated between 120A & 225A. Find the direct and quadrature axis reactances per phase. Neglect R_a .
[EM-II, Oct/Nov-2016, Reg, Set-2]

Sol: Given data

$$V_{\min} = 1196 \text{ V (line)} = 690.53 \text{ V (ph)}$$

$$V_{\max} = 1217 \text{ V (line)} = 702.66 \text{ V (ph)}$$

$$I_{\min} = 120 \text{ A}$$

$$I_{\max} = 225 \text{ A}$$

from Slip Test

$$\text{Direct axis reactance / ph} = \frac{V_{\max}}{I_{\min}} = \frac{702.66}{120}$$

$$X_d = 5.86 \Omega$$

$$\text{Quadrature axis reactance / ph} = \frac{V_{\min}}{I_{\max}} = \frac{690.53}{225}$$

$$X_q = 3.07 \Omega$$

Prob: A 5KVA, 220V, star connected 3- ϕ salient pole alternator with direct axis and quadratic axis reactances of 12Ω & 7Ω respectively, delivers full load current unity power factor. Calculate the excitation voltage. Neglect armature resistance.

Sol: Given Data Power rating = 5KVA
 Voltage, $V = 220V (L) = 127V (Ph)$
 $X_d = 12\Omega$, $X_q = 7\Omega$, $R_a = 0$.

$$\text{Full load current } I = \frac{5K}{\sqrt{3} \times 220} = 13.12A \text{ at UPF}$$

$$\text{Load power factor, } \cos\phi = 1, \Rightarrow \phi = 0^\circ$$

$$\therefore \text{Internal power factor angle, } \theta = \tan^{-1} \left[\frac{V \sin\phi + I X_q}{V \cos\phi + I R_a} \right]$$

$$\therefore \theta = \tan^{-1} \left[\frac{127 \times 0 + 13.12 \times 7}{127 \times 1 + 0} \right] = 35.87^\circ$$

$$\therefore \text{Excitation Voltage, } E_o = V \cos\alpha + I R_a \cos\phi + I_d X_d$$

$$\alpha = \theta - \phi = 35.87^\circ - 0^\circ = 35.87^\circ$$

$$I_d = I \sin\theta = 13.12 \times \sin(35.87^\circ) = 7.69A$$

$$\therefore E_o = 127 \times \cos(35.87^\circ) + 0 + 7.69 \times 12 = 195.2V$$

$$\boxed{E_o = 195.2V}$$

Parallel Operation of Synchronous Generators

* Necessity of parallel operation of Alternators:

1. Local or regional power use may exceed the power of a single available generator.
2. Parallel alternators allow one or more units to be shut down for scheduled or emergency maintenance while the load is being supplied with power.
3. Generators are inefficient at part load, so shutting down one or more generators allows the remaining load to be carried with less machines that are efficiently loaded.
4. Load growth can be handled by added machines without disturbing the original installation.
5. Available machine prime-movers and generators can be matched for economic first cost and flexible use.

* Requirements for paralleling:

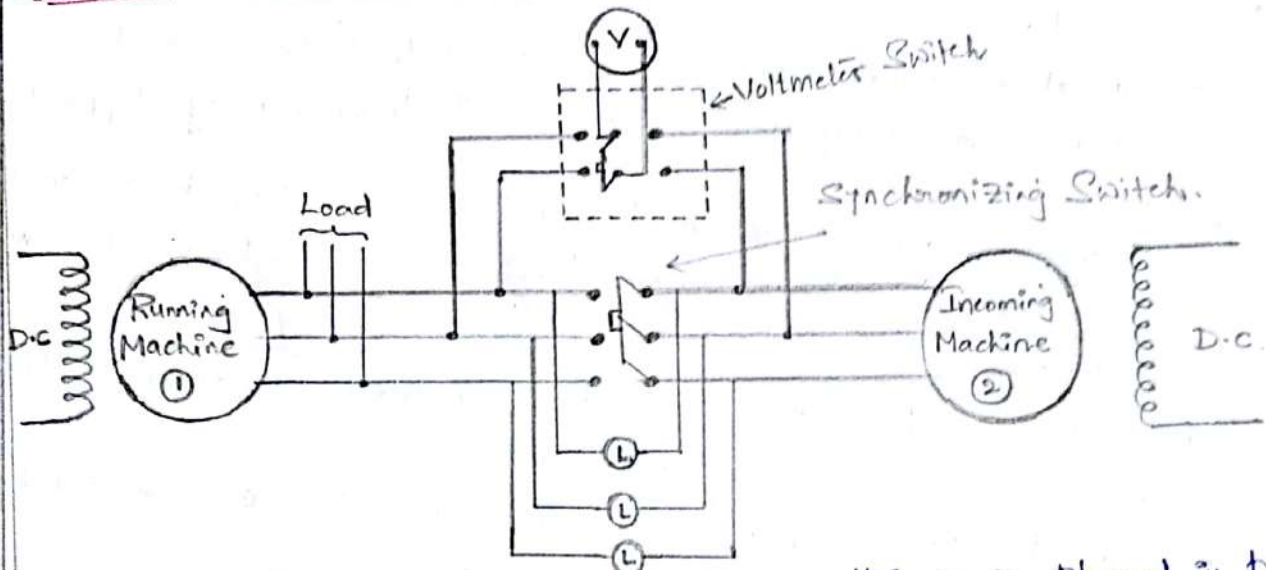
The requirements for paralleling include the requirements for D.C. machines plus a few others.

1. The voltages must be the same at the paralleling or junction even though not the same at the alternators.
2. The phase sequence for three phase must be the same at the paralleling point.
3. The incoming machine must be in phase at the moment of paralleling. It will stay in phase under normal conditions after paralleling. It is important to recognize that phase sequences and in phases are not the same thing.
4. The line frequencies must be identical at the paralleling point. In the vast majority of cases, this means the same frequency at the generator because frequency changing is not economic. Mixed frequencies must be paralleled through some frequency conversion means for compatibility at the point of interaction.
5. The prime-movers must have relatively similar and drooping speed load characteristics. This is to prevent a machine with a rising speed load characteristics from taking more and more of the

load until it fails from overload.

Violation of these requirements for paralleling would result in circulating currents between the machines varying from uneconomic, to serious, to disastrous.

* Alternator Synchronising Procedure:



It may logically be assumed that one alternator is placed in parallel with one or more other alternators only when additional load requires it. Those alternators already carrying load are known as the running machines, while that which is to be placed in the system is known as the incoming machine. At the time of synchronizing, the following conditions must be met.

1. The effective Voltage of the incoming alternator must be exactly equal to that of the others or of the bus-bars connecting them.
2. The phase rotation, or sequence of the running and incoming alternators must be the same.
3. The individual phase Voltages which are to be connected to each other must be in exact phase opposition. This is the same as saying that D.C. generators must be connected + to + & - to -.
4. The frequencies should be the same, although it is more desirable that the frequencies at the instant of paralleling be almost, but not quite, identical.

Synchronizing Lamps:

1. To satisfy the first condition of paralleling a voltmeter can be used
2. Satisfaction of the other conditions of phase sequence, voltage oppo

Sition, and frequency may be determined by the use of the incandescent lamps connected b/w the two machines as shown in fig.

At any instant it is seen, that the Voltage across the lamp is the Sum of the individual Phase Voltages.

The procedural steps for putting incoming alternator in parallel with the running machine as follows:

Step 1: The primemover of the incoming machine is started, and the alternator is brought up to near its rated speed.

Step 2: By adjusting the field current, the terminal Voltage of incoming Voltage machine is made the same as that of the running alternator.

The lamp in the circuit will now flicker at a rate equal to the difference in frequency of the two alternators. If the phases are properly connected, all lamps will be bright and dark at the same time. If this is not the case, then it means that the phase sequences are not correct, and it is merely necessary to interchange two of the lines leads of the incoming machine.

Step 3: Further adjustment of the incoming primemover is now necessary, until the lamps flicker at a very low rate, usually less than one dark period per second.

Step 4: Final adjustment of the incoming Voltage again made and the synchronizing Switch is thrown in the middle of a dark period. The Voltage across the lamps varies from zero to twice the phase Voltage, and therefore, the lamps must be rated for this higher Voltage. It is

The lamp methods has the following advantages and disadvantages:

Advantages:

- (i) Equipment is inexpensive
- (ii) proper phase sequence is readily obtained.

Disadvantages:

- (i) Lamps go dark at somewhat less than half their rated Voltage, and so the synchronizing Switch might be closed when there is a considerable phase difference between the machines, with a high circulating current resulting in

Possible damage to the machines.

(i) The lamp filaments can burn out.

(iii) The flicker of the lamps does not indicate which machine has the higher frequency.

There are two other lamp methods which are given below:

(i) If the lamps are connected across the phases, that is p to ϕ' , ϕ to R' , R to p' , they will again flicker in unison. This time, however, the proper synchronizing moment occurs when all three are in the middle of a bright period. This has the advantage of avoiding synchronizing when the lamps may have burned out, but it is more difficult to estimate the middle of a bright period than the middle of a dark one.

(ii) In 3rd lamp method, known as the "two bright, one dark method", only two of the lamps are cross connected. Thus the connections are phase p to p' , ϕ to R' and R to ϕ' . This method supposedly avoids the disadvantages of both previous lamp methods, but actually may be more confusing to the eye.

In case of high voltage alternators, the lamps are connected through transformers to obtain the nominal voltages for which lamps are rated.

Synchroscope Synchronizing:

In large central station installations, an additional device - called a "Synchroscope" is used.

A synchroscope has a rotating hand and a dial labelled with slow and fast direction arrows to show the incoming machine speed relation. In addition, an index point shows the actual in phase voltage position. During synchronization, as the incoming machine rotational speed approaches near synchronism, the speed of the synchroscope hand drops enough to become visible. The hand speed is proportional to the difference in speed. The slow indication is accompanied with an arrow -

Showing that Counter clockwise hand rotation means below Synchronous Speed. Similarly clockwise rotation means above Speed. When the Speeds are matched as that the hand Speed is very slow, the hand is matched until it points to the index mark, where upon the paralleling switch may be closed.



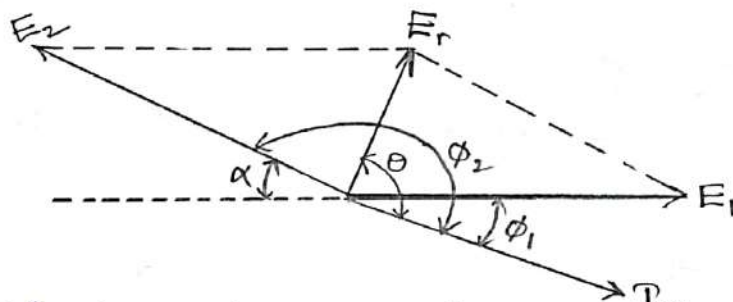
* Synchronising Current, power and Torque:

Synchronising Current:

→ When in exact Synchronism, the two alternators have equal induced emfs and are in exact phase opposition.



→ When the induced emf's of the two alternators are equal but not in exact phase opposition, their resultant emf acts around the local circuit and causes flow of current called the "Synchronising current".



Let - $(180^\circ - \alpha)$ = angular phase difference b/w the two induced emfs E_1 and E_2 ; E = each emf (phase emf).

$$\text{Resultant emf, } E_r = 2E \cos \left[\frac{180^\circ - \alpha}{2} \right] = 2E \cos \left[90^\circ - \frac{\alpha}{2} \right] = 2E \sin \frac{\alpha}{2}$$

$$= 2E \times \frac{\alpha}{2} \quad [\because \alpha \text{ is very small}]$$

$$E_r = E\alpha$$

$$\text{Synchronising current, } I_{sy} = \frac{E_r}{Z} = \frac{E\alpha}{Z} \text{ lagging behind the } E_r \text{ by } \theta$$

$$\theta = \tan^{-1} \left(\frac{X_s}{R} \right)$$

Where Z → Combined impedance of two alternators/ph (or) alternator impedance if alternator connected to busbar

→ If resistance is very small as compared to synchronous reactance of the alternator, then

$$\text{Synchronising current, } I_{sy} = \frac{E\alpha}{X_s} \text{ lagging behind } E_r \text{ by } 90^\circ$$

Synchronising Power:

$$\text{Power Supplied by machine-1} = E_1 I_{sy} \cos \phi_1$$

$$\text{Power Supplied by machine-2} = E_2 I_{sy} \cos \phi_2$$

& Also, Power Supplied by machine-1

= Power Supplied by machine-2 + Copper losses.

$$E_1 I_{sy} \cos \phi_1 = E_2 I_{sy} \cos \phi_2 + \text{Copper losses.}$$

Power Supplied by machine-1 is called Synchronising Power,

$$P_{sy} = E_1 I_{sy} \cos \phi_1 = E_1 I_{sy} \quad [E_1 = E \text{ and } \phi_1 \text{ is very small}]$$

$$P_{sy} = E \times \frac{E \kappa}{X_s} = \frac{E^2 \kappa}{X_s}$$

$$P_{sy} = \frac{\kappa E^2}{X_s}$$

$$\text{Total Synchronising power for 3-}\phi = 3P_{sy} = \frac{3\kappa E^2}{X_s}$$

Synchronising Torque:

If T_{sy} is the Synchronising torque in N-m, the total synchronising power

$$3P_{sy} = T_{sy} \times \frac{2\pi N_s}{60}$$

$$T_{sy} = \frac{3P_{sy} \times 60}{2\pi N_s}$$

Where N_s is Synchronous Speed in rpm = $\frac{120f}{P}$

Prob: A 3MVA, 6-pole alternator runs at 1000 rpm in parallel with other machines on 33kV busbars. The Synchronous reactance is 20%. Calculate the synchronising power per one mechanical degree of displacement and the corresponding synchronising torque.

Sol: Given data Rating, $S = 3\text{MVA}$

Poles, $P = 6$

Speed, $N_s = 1000\text{ rpm}$

Voltage, $E = 3.3\text{ kV (line)} = 1905\text{ V (ph)}$

$X_s = 20\%$

$$\text{full load current, } I = \frac{S}{\sqrt{3}V} = \frac{3 \times 10^6}{\sqrt{3} \times 3.3k} = 524.9A$$

$$\text{Synchronous Impedance, } Z_s = \frac{1905}{524.9} = 3.63\Omega$$

$$\text{Synchronous Reactance, } X_s = 20\% \text{ of } Z_s = 0.726\Omega$$

$$\alpha = 1^\circ (\text{Mech}) = 1^\circ \times \frac{P}{2} (\text{elec}) = 1^\circ \times \frac{6}{2} = 3^\circ (\text{elec}) = \frac{\pi}{60} \text{ rad}$$

$$\text{Synchronising power, } P_{sy} = \frac{\alpha E^2}{X_s} = \frac{(3 \times \frac{\pi}{180}) \times 1905^2}{0.726} = 261.73 \text{ kW per phase}$$

$$\text{Total Synchronising power, } 3P_{sy} = 3 \times 261.73 \text{ kW} = 785.19 \text{ kW}$$

$$\text{Synchronous Speed, } N_s = 1000 \text{ rpm}$$

$$\text{Synchronising Torque, } T_{sy} = \frac{3P_{sy}}{\omega_s} = \frac{785.19k}{2\pi \times \frac{1000}{60}} = 7498.01 \text{ N-m}$$

Prob: A 4500KVA, 50Hz, 3- ϕ , Synchronous generator having a Synchronous reactance of 0.3pu is running at 1500rpm and is excited to give 11000V. If the rotor deviates slightly from its equilibrium position, what is the synchronizing torque in N-m per degree mechanical displacement.

801: Given data rating, $S = 4500 \text{ KVA}$

$$\text{frequency } f = 50 \text{ Hz}$$

$$\text{Synchronous Reactance, } X_s = 0.3 \text{ pu}$$

$$\text{Synchronous Speed, } N_s = 1500 \text{ rpm} \Rightarrow \text{Poles, } P = 4$$

$$\text{Voltage, } E = 11 \text{ kV (line)} = 6351.04 \text{ V (ph)}$$

$$\text{full load current, } I = \frac{S}{\sqrt{3}V} = \frac{4500k}{\sqrt{3} \times 11k} = 236.2 \text{ A}$$

$$\text{Synchronous Reactance, } X_s = 0.3 \times \frac{6351.04}{236.2} = 8.07\Omega$$

$$\alpha = 1^\circ (\text{Mech}) = 1^\circ \times \frac{P}{2} = 1 \times \frac{4}{2} = 2^\circ (\text{elec}) = 2^\circ \times \frac{\pi}{180} = \frac{\pi}{90}$$

$$\text{Total synchronizing power, } 3P_{sy} = \frac{3\alpha E^2}{X_s} = \frac{3 \times \frac{\pi}{90} \times 6351.04^2}{8.07}$$

$$3P_{sy} = 523.41 \text{ kW}$$

$$\text{synchronizing torque, } T_{sy} = \frac{3P_{sy}}{\omega_s} = \frac{523.41k}{2\pi \times \frac{1500}{60}}$$

$$T_{sy} = 3332.1 \text{ N-m}$$

Prob. 3MVA, 6-pole alternator runs at 1000 rpm on 3.3KV busbars. The synchronous reactance is 25%. Calculate the synchronising power and torque per mechanical degree of displacement when alternator is supplying full load at 0.8 power factor lagging.

Sol. Given data Rating, $S = 3\text{MVA}$

poles, $P = 6$

Synchronous Speed, $N_s = 1000\text{ rpm}$

Voltage, $E = 3.3\text{KV (line)} = 1905.3\text{V (ph)}$.

Synchronous reactance, $X_s = 25\%$

full load power factor, $\cos\phi = 0.8\text{ lag}$.

$$\alpha = 1^\circ (\text{mech}) = 1^\circ \times \frac{6}{2} = 3^\circ (\text{elec}) = \frac{\pi}{60} \text{ rad.}$$

$$\text{full load current, } I = \frac{S}{\sqrt{3}V} = \frac{3 \times 10^6}{\sqrt{3} \times 3.3\text{k}} = 577.4\text{A}$$

Voltage at 0.8 pf lag

$$E_0 = \sqrt{(V \cos\phi)^2 + (V \sin\phi + I X_s)^2}$$

$$Z_s = \frac{E}{I} = \frac{1905.3}{577.4} = 3.299\Omega, X_s = \frac{25}{100} \times 3.299 = 0.825\Omega$$

$$E_0 = \sqrt{(1905.3 \times 0.8)^2 + (1905.3 \times 0.6 + 577.4 \times 0.825)^2}$$

$$E_0 = 2224\text{V}$$

$$\begin{aligned} \text{Synchronising power, } 3P_{sy} &= \frac{3 \times E_0 \times E}{X_s} \\ &= \frac{3 \times \frac{\pi}{60} \times 2224 \times 1905.3}{0.825} = 806.796\text{ kW} \end{aligned}$$

$$\begin{aligned} \text{Synchronising Torque, } T_{sy} &= \frac{3P_{sy}}{\omega_s} = \frac{3P_{sy}}{\frac{2\pi N_s}{60}} \\ &= \frac{806.796\text{ K}}{\frac{2\pi \times 1000}{60}} = 7704.34\text{ N-m.} \end{aligned}$$

Prob. A 2MVA, 3- ϕ , γ -connected, 4-pole, 750 rpm alternator is operating at 6000V busbars. The Sync. reactance is 6 Ω /ph. Find Synchronising power & Torque for full load 0.8 pf lag. [Oct/Nov-16, EH-II, Supply, Set-3, 5(b)].

Prob. A 3- ϕ , 3000KVA, 6-pole, 50Hz alternator running in parallel with 3.3KV busbar. $Z_s = 25\%$. Calculate the Sync. power & torque for one mech. degree of displacement. [Oct/Nov-16, EH-II, Supply, Set-4, 5(b)].

* Effect of Reactance:

It has been observed that without reactance, the synchronous machines cannot develop motor and generator power, respectively to restore synchronism and consequently parallel operation is not possible.

* Effect of increasing the excitation of one of the Alternator:

* If the excitation of an alternator operating in parallel with other alternators is increased above its normal value of excitation, its power factor changes in the lagging direction, and its current output increases with no appreciable change in its kW load.

When the alternator is under-excited its power factor becomes more leading and its current output increases with no change in kW output. This increase in current in either case is not applicable to the load but circulates between the alternators connected to the system, thereby increasing their losses and reducing their useful capacity. It is desirable in most cases, therefore, to operate at the same factor (at near the load p.f.), keeping the circulating currents to the minimum.

* The proper excitation for alternators operating in parallel, in general, is the amount of excitation each alternator would need if it were carrying its load alone at the same frequency and voltage.

* The voltage of the system in which several alternators are operating in parallel may be raised or lowered by varying the excitation of all the machines up or down as needed; this is accomplished automatically through the voltage regulators.

* Effect of increasing the driving torque of one of the Alternator

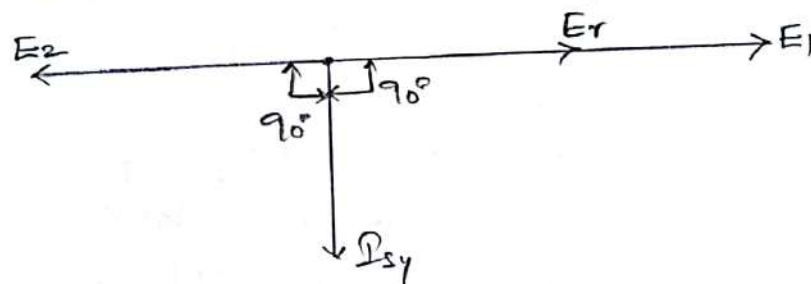
* When two alternators are running in parallel and if the driving torque of one of the alternators is increased, it is further loaded and other is relieved of its load. If the output of the alternator, whose driving torque has been increased, becomes more than the total load supplied, then the other alternator runs as a synchronous motor.

* Effect of Changes in Speed of one of the Alternators:

- * When in exact Synchronism, the two alternators have equal induced emfs and are in exact phase opposition. No Circulating Current flows round the local circuit.
- * When the Speed of Alternator 2 is reduced, E_2 falls back by a phase angle of α (elec), the magnitude of induced emf's, E_1 & E_2 remains the same. It will cause resultant emf E_r acting in the local circuit, which will cause a flow of Synchronising current I_{sy} in the local circuit. This I_{sy} sets up a synchronising torque tending to retard the generating alternator 1 and accelerating the motoring alternator 2. Thus the two alternators come again in Synchronism.
- * Thus by increasing or reducing the speed of all the alternators simultaneously, the frequency of the system can be raised or lowered.

* Effect of unequal Voltages:

- * Let there to be two alternators having emfs E_1 & E_2 ($E_1 > E_2$) exactly in phase opposition relative to external load circuit. The resultant emf (E_r) equal to $E_1 - E_2$ acts in the local circuit & causes Synchronising current I_{sy} around the local circuit. The current I_{sy} lags behind E_r or E_1 by 90° and leads E_2 by 90° . Thus current I_{sy} produces demagnetising effect on M/C No. 1, resulting thereby reduction in E_1 and magnetising effect on M/C No. 2, resulting thereby increase in E_2 . Thus difference of E_1 & E_2 is reduced and stable condition is established.



* Distribution of load:

- (i) Effect of change in driving torque
- (ii) Effect of change in Excitation.

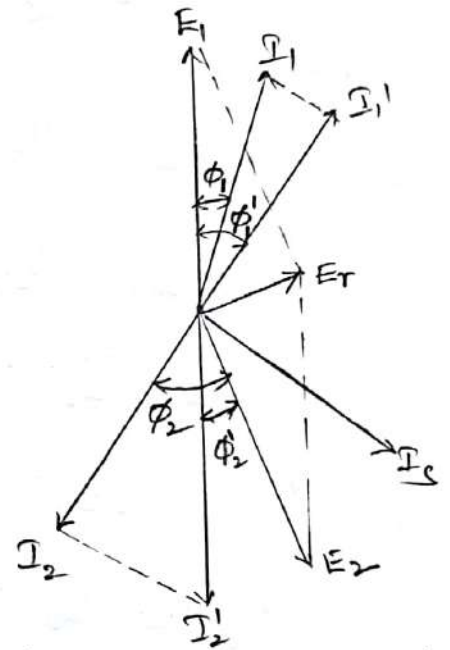
It will be shown that:

- (i) Amount of load taken up by an alternator running in parallel with other machines is solely determined by its driving torque i.e., by the power input to its prime-movers.
- (ii) Any change in its excitation merely changes its kVA output but not its kW output. In other words, it merely changes the power factor at which load is delivered.

Let us consider two 1- ϕ alternators running in parallel with generated emf's E_1 & E_2 respectively. If they are in phase with external circuit, then they will be in direct opposition with respect to their local circuit.

Let their excitations and power inputs to their prime-movers are changed such that they deliver currents of I_1 & I_2 at respective power factors of $\cos \phi_1$ & $\cos \phi_2$. Then, total current supplied to the load equal to vector sum of I_1 & I_2 .

Consider that the power input to the prime-mover of the 2nd machine is increased. Then its emf vector swings ahead by a certain angle. Now, a resultant voltage E_r or E_s is produced which, acting on the local circuit, sets flowing a current I_s which lags almost by 90° behind E_r . The new armature current I_2' is greater than I_2 and its is less -



lagging than it. Hence, the 2nd alternator takes up increased share of the load. This synchronising current is taken up by the first machine. Its armature current also changes from I_1 to I_1' . It may be noted that I_1' is lesser and is at a greater angle from E_1 . Hence, this alternator is taking up a lesser share of its load.

Thus we find that by increasing the input to its prime-mover, an alternator can be made to take a greater share of load, though at different power factor.

The following points are worth noting:

- * If excitation is changed, merely power factor at which load is delivered is changed.
- * The load taken up by an alternator directly depends upon its driving torque (or angular advance of the rotor).
- * By keeping the input to the prime mover of an alternator constant, if excitation is changed, then only KVA component of the o/p is changed and not KW.

* Load Sharing between Two Alternators:

Let us consider two machines with identical speed characteristics running in parallel with a common terminal voltage V and load impedance Z .

$$I = I_1 + I_2 = \frac{V}{Z}$$

from figure

$$I_1 = \frac{E_1 - V}{Z_1}, \quad I_2 = \frac{E_2 - V}{Z_2}$$

$$I = \frac{E_1 - V}{Z_1} + \frac{E_2 - V}{Z_2}$$

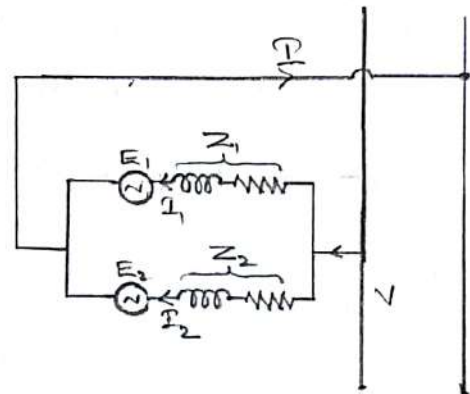
$$I = \frac{E_1}{Z_1} + \frac{E_2}{Z_2} - V \left[\frac{1}{Z_1} + \frac{1}{Z_2} \right]$$

$$V = IZ = \left[\frac{E_1}{Z_1} + \frac{E_2}{Z_2} \right] Z - V \left[\frac{1}{Z_1} + \frac{1}{Z_2} \right] Z$$

$$\therefore V \left[1 + \frac{Z}{Z_1} + \frac{Z}{Z_2} \right] = \left[\frac{E_1}{Z_1} + \frac{E_2}{Z_2} \right] Z$$

$$\Rightarrow V \left[\frac{1}{Z} + \frac{1}{Z_1} + \frac{1}{Z_2} \right] = \left[\frac{E_1}{Z_1} + \frac{E_2}{Z_2} \right]$$

$$\therefore V = \frac{\left[\frac{E_1}{Z_1} + \frac{E_2}{Z_2} \right]}{\frac{1}{Z} + \frac{1}{Z_1} + \frac{1}{Z_2}}$$



Prob. Two 1- ϕ alternators operate in parallel and supply a load impedance of $(3+j4)\Omega$. If the impedance of each machine is $(0.2+j2)\Omega$ and emfs are $(200+j0)$ and $(220+j0)V$ respectively, determine for each machine:

(i) Terminal Voltage (ii) power factor and (iii) output.

Sol. Given data Load Impedance, $Z_L = (3+j4)\Omega$
 Alt -1 Impedance, $Z_1 = (0.2+j2)\Omega$
 Alt -2 Impedance, $Z_2 = (0.2+j2)\Omega$
 Alt -1 EMF, $E_1 = (200+j0)V$
 Alt -2 EMF, $E_2 = (220+j0)V$.

Terminal Voltage:

$$V = E_1 - I_1 Z_1 \quad \text{or} \quad V = \frac{E_1 + \frac{E_2}{\frac{1}{Z_L} + \frac{1}{Z_1} + \frac{1}{Z_2}}}{\frac{1}{Z_L} + \frac{1}{Z_1} + \frac{1}{Z_2}}$$

$$\therefore V = \frac{\frac{200}{0.2+j2} + \frac{220}{0.2+j2}}{\frac{1}{3+j4} + \frac{1}{0.2+j2} + \frac{1}{0.2+j2}} = \frac{208.958 \angle -1.47^\circ}{1.17 \angle -1.383^\circ} = 178.48 \angle -5.08^\circ V$$

$$V = 178.48 \angle -5.08^\circ V$$

Machine-I:

$$\text{Current } I_1 = \frac{E_1 - V}{Z_1} = \frac{200 \angle 0^\circ - 178.48 \angle -5.08^\circ}{0.2+j2} = 13.56 \angle -48.87^\circ A$$

$$\therefore \text{Power factor} = \cos \phi_1 = \cos (-48.87^\circ) = 0.7218 \text{ lag.}$$

$$\phi_1 = -48.87^\circ - (-5.08^\circ) = -43.79^\circ$$

$$\text{Power output} = V I_1 \cos \phi_1 = 178.48 \times 13.56 \times 0.7218 = 1747 W$$

$$O/p = 1747 W$$

Machine-II:

$$\text{Current } I_2 = \frac{E_2 - V}{Z_2} = \frac{220 \angle 0^\circ - 178.48 \angle -5.08^\circ}{0.2+j2} = 22.4 \angle -63.8^\circ A$$

$$\phi_2 = -63.8^\circ - (-5.08^\circ) = -58.72^\circ$$

$$\text{Power factor, } \cos \phi_2 = \cos (-58.72^\circ) = 0.5192 \text{ (lag)}$$

$$\text{Power output} = V I_2 \cos \phi_2 = 178.48 \times 22.4 \times 0.5192 = 2075 W$$

$$O/p = 2075 W$$

Prob: Two 3- ϕ alternators are working in parallel with the following particulars:

Alternator 1: $Z_1 = (0.2 + j2) \Omega/\text{ph}$; $E_1 = (2000 + j0) \text{ V/ph}$

Alternator 2: $Z_2 = (0.2 + j2) \Omega/\text{ph}$; $E_2 = (2200 + j100) \text{ V/ph}$

Load $Z_L = (3 + j4) \Omega/\text{ph}$. Determine the KW output and power factor of each alternator.

Prob: Two identical 3- ϕ alternators work in parallel and supply a total load of 1500KW at 11KV at a power factor of 0.867 lag. Each machine supplies half the total power. The synchronous reactance of each is 50 Ω per phase and the resistance is 4 Ω per phase. The field excitation of first machine is so adjusted that its armature current is 50A lagging. Determine the armature current of the 2nd alternator and the generated voltage of the first machine.

Prob Two synchronous generators are connected to bus bars having a constant voltage of $10000 \angle 0^\circ \text{ V}$. Generator A has an induced emf of $13000 \angle 22.6^\circ \text{ V}$ and is reactance of 2Ω ; generator B has an emf of $12500 \angle 36.9^\circ \text{ V}$ and a reactance of 3Ω . Find the current, KW & KVAR supplied by each generator.

Sol: Given data Terminal Voltage, $V = 10 \text{ KV} \angle 0^\circ$
 Alternator-1; $E_1 = 13000 \angle 22.6^\circ \text{ V}$, $X_{s1} = 2\Omega$
 Alternator-2; $E_2 = 12500 \angle 36.9^\circ \text{ V}$, $X_{s2} = 3\Omega$

From the parallel operation of two Alternators

$$V = E_1 - I_1 Z_1 = E_1 - j I_1 X_{s1}$$

$$V = E_2 - I_2 Z_2 = E_2 - j I_2 X_{s2}$$

$$\therefore I_1 = \frac{E_1 - V}{j X_{s1}} = \frac{13000 \angle 22.6^\circ - 10000 \angle 0^\circ}{j 2} = 2690.97 \angle -21.83^\circ \text{ A}$$

$$I_2 = \frac{E_2 - V}{j X_{s2}} = \frac{12500 \angle 36.9^\circ - 10000 \angle 0^\circ}{j 3} = 2501.75 \angle 0.03^\circ \text{ A}$$

Generator 1:

$$I_1 = 2690.97 \angle -21.83^\circ \text{ A}$$

$$\text{Generator 1 Output, } S = V I_1 = 10 \text{ K} \times 2690.97 = 26.91 \text{ MVA}$$

$$\text{Power factor} = \cos \phi_1 = \cos (-21.83^\circ) = 0.928 \text{ lead.}$$

$$\text{KW rating, } P = S \cos \phi_1 = 24.97 \text{ KW}$$

$$\text{KVAR rating, } Q = S \sin \phi_1 = -10.00 \text{ MVAR.}$$

Generator -2:

$$I_2 = 2501.75 \angle 0.03^\circ \text{ A}$$

$$\text{Apparant power, } S = V I_2 = 10 \text{ kV} \times 2501.75 = 25.02 \text{ MVA}$$

$$\text{Power factor, } \cos \phi_2 = \cos(0.03) = 0.99 \text{ lag}$$

$$\text{Active power, kW rating} = S \cos \phi_2 = 25.01 \text{ MW}$$

$$\text{kVAR rating, } Q = S \sin \phi_2 = 13.10 \text{ kVAR}$$

Prob Two alternators working in parallel supply a lighting load of 3000kW and a motor load aggregating to 5000kW at a P.f. of 0.72. One machine is loaded up to 5000kW at 0.8 pf lag. What is the load and power factor of the other machine? [Ans: 0.942 lag]

Prob: The following loads are supplied by two alternators running in parallel:

- (i) 1400KW at pf 0.86 lag
- (ii) 900KW at p.f 0.8 lagging.
- (iii) 800kw at pf unity
- (iv) 500KW at pf 0.8 leading.

If the load on the machine is adjusted to 2100KW at pf 0.92, find the load and pf of the other machine. [Ans: 0.9878 lag].

Synchronous Motors

* The Synchronous motor is the one type of 3-phase AC motor which operates at a constant speed from no-load to full-load. It is similar in construction to 3-ph AC generator in that it has a revolving field which must be separately excited from a D.C. supply. By changing the D.C. field excitation current, the power factor of this motor can be varied over a wide range of lagging and leading values.

* This motor is used in many individual applications because of its fixed speed from no-load to full-load, its high efficiency and initial cost. It is also used to improve power factor of 3- ϕ AC industrial ckt's.

* Characteristic features, Advantages and Disadvantages:

Characteristic features:

1. It runs either at synchronous speed or not at all. The speed can be changed by changing the frequency only ($\because N_s = 120f/p$).
2. It is not inherently self-starting. It has to be run up to synchronous or near synchronous speed by some means before it can be synchronized to the supply.
3. It can be operate under a wide range of power factor both lagging and leading.
4. On no-load the motor draws very low current from supply to meet the internal losses. With fixed excitation the input current increases in load is possible. If the motor is further loaded, the motor will stop.

Advantages:

1. These motors can be used for power factor correction.
2. They are more efficient than induction motors of corresponding output (kW) and voltages rating.
3. They may be less costly for the same output, speed, and voltage ratings as compared to induction motors.
4. They give constant speed from no-load to full load.
5. Electromagnetic power varies linearly with the voltage.

Disadvantages:

1. They require D.C. excitation which must be supplied from external source.
2. They have a tendency to hunt.
3. They cannot be used for variable speed jobs as speed adjustment cannot be done.
4. They require collector rings and brushes.
5. They cannot be started under load. Their starting torque is zero.
6. They may fall out of synchronism and stop when overloaded.

* Applications: used in

1. power houses and substations.
2. Factories
3. Mills-industries etc.
4. For constant speed equipments. (Fans, Blowers, centrifugal pumps, Motor generator sets, ammonia & air compressors etc.)

* Construction of 3- ϕ Synchronous Motor:

A 3- ϕ synchronous motor consists of the following essential parts:

1. Laminated stator core with 3- ϕ armature winding.
2. Revolving field complete with damper windings and slip rings
3. Brushes and brush holders.
4. Two end shields to house the bearings that support the shaft.

* The stator core and windings of a sync. motor are similar to those of a 3- ϕ squirrel cage induction motor or a wound rotor induction motor. The leads for the stator winding, marked T_1, T_2 & T_3 , terminate in a terminal box usually mounted on the side of the motor frame.

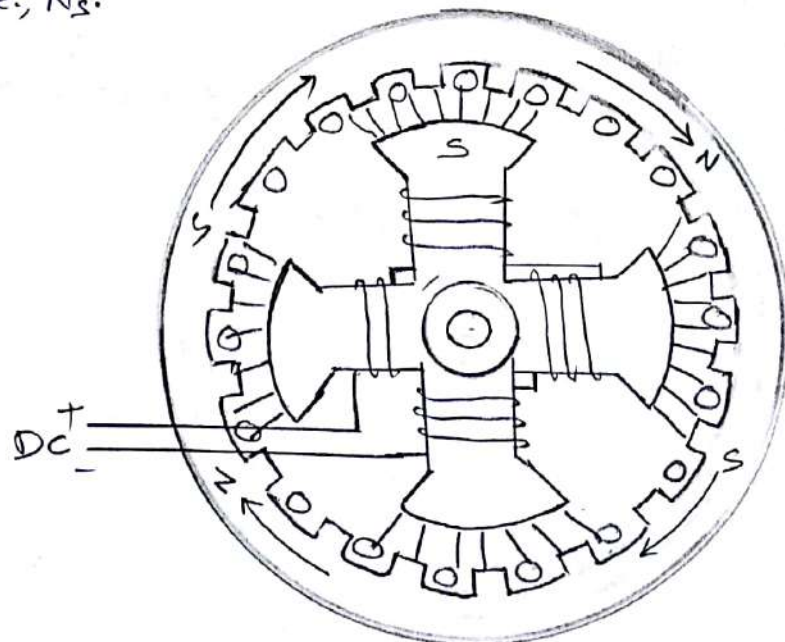
* The rotor is generally a salient pole rotor. The no. of rotor field poles must equal the no. of stator field poles. In order to eliminate the hunting and to develop the necessary starting torque when A.C. voltage is applied to the stator, the rotor poles contain pole-face conductors which are short circuited at their ends. This damper winding consists of solid copper bars embedded at the surface of the pole face & short circuited at each end by means of a shorting strip.

* The field leads are brought out to two slip rings mounted on the rotor shaft. Carbon brushes mounted in brush holders make contact with two slip rings. The terminals of the field circuit are brought out from the brush holders to a second terminal box mounted on the motor frame. The two leads for the field circuit are marked F_1 & F_2 .

* Principle of operation:

When the stator windings of a 3- ϕ synchronous motor are supplied with rated 3- ϕ voltage, a rotating field travelling at synchronous speed is set up. The synchronous speed is found from the relation, $N_s = 120f/p$. This rotating magnetic field cuts the squirrel cage winding of the rotor and induces voltages & currents in the bars of this winding. The resultant magnetic field of squirrel cage winding embedded in the rotor field poles reacts with the stator field in such a manner as to cause the rotation of the motor. The rotor will increase its speed to a point slightly below the synchronous speed of the stator field. The rotor of the typical synchronous motor accelerates to about 95 to 97% of synchronous speed when started as an induction motor with damper windings. The field circuit is now excited from an outside source of D.C and magnetic poles of fixed polarity are set up in the rotor field cores. The fixed magnetic poles of the rotor are attracted to unlike poles of the rotating magnetic field set up by the stator windings.

The fig shows the locking of rotor field poles with unlike poles of stator field. The rotor then runs at the same speed as that of the stator field i.e., N_s .



* Starting Methods of 3- ϕ Synchronous Motor:

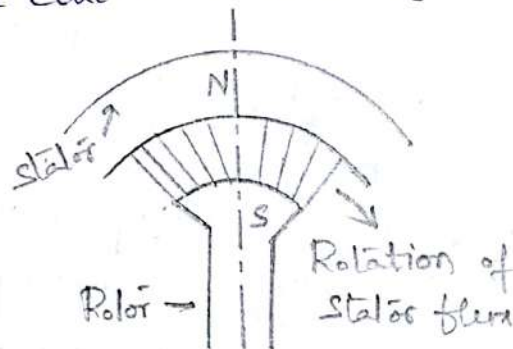
1. A DC motor coupled to the Synchronous motor shaft.
2. An AC motor coupled to the Synchronous motor shaft.
3. A small induction motor of at least one pair of poles less than the sync. motor. (using pony motors).
4. Using the damper windings as a Squirrel Cage induction motor.

[Explain each method in detail in the examinations.]

* Effect of Load on a Synchronous Motor:

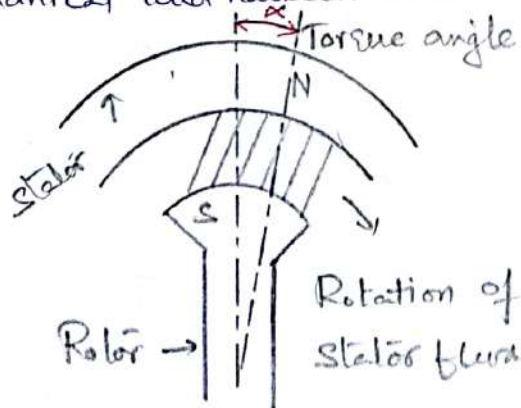
When mechanical load on a DC motor or an AC motor is increased, the speed decreases. This, in turn, decreases the back emf or counter emf (E_b) so that the source is able to supply more current to meet the increased load demand. However, this action cannot take place in the synchronous motor for the rotor must run at synchronous speed at all loads.

The fig(1) shows the relative position of a stator and rotor pole at no-load, pole centres are directly in line with each other.



Fig(1). Torque angle at no load ($\alpha = 0^\circ$)

The fig(2) represents the relative position of the stator and the rotor poles after mechanical load has been added to the motor.



Fig(2). Torque angle at loaded load ($\alpha \neq 0^\circ$)

Now there has been a shift of the rotor pole in a direction opposite to that of the stator field flux and the direction of the rotor. It may be kept in mind that there has been no change in speed as the rotor will continue to rotate at Synchronous Speed. There is only an angular displacement between the centres of the stator and rotor field poles. The angular displacement is called the "torque angle."

No load Condition Vector Diagram:

When the motor is running on no-load and is having no losses, $V = E_b$, hence their vector difference is zero and so is the armature current. Motor intake is zero, as there is neither load nor losses to be made by it. In other words, the motor just floats.

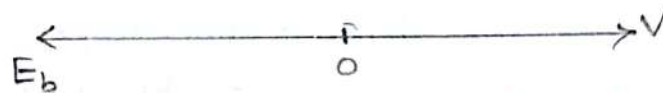


Fig. No-load (no losses).

When the motor is on No-load but has losses. The vector for E_b falls back by a certain angle α_0 (No-load torque angle), so that a resultant voltage E_r and hence current I_0 brought into existence which supplies losses.

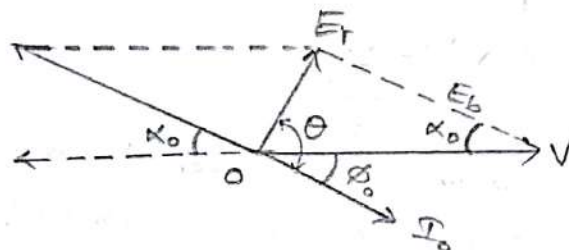


Fig. No-load (With losses).

Load Condition Vector Diagram:

When the motor is loaded, it slows down momentarily to adjust itself to the change in load condition, so the rotor pole falls back a little more relative to the stator pole, as shown in fig. Hence the torque angle increases with the increase in load. Due to increase in load or torque angle α , the resultant voltage E_r across the armature circuit increases, and, therefore, current drawn from the supply mains increases. Thus a motor is able to supply increased mechanical load, not by reduction in speed,

but by shift in relative positions of the stator and rotating magnetic field (or stator flux). From the phasor diagram it is obvious that for increasing load with a constant value of back emf E_b , the phase angle ϕ increases in lagging direction.

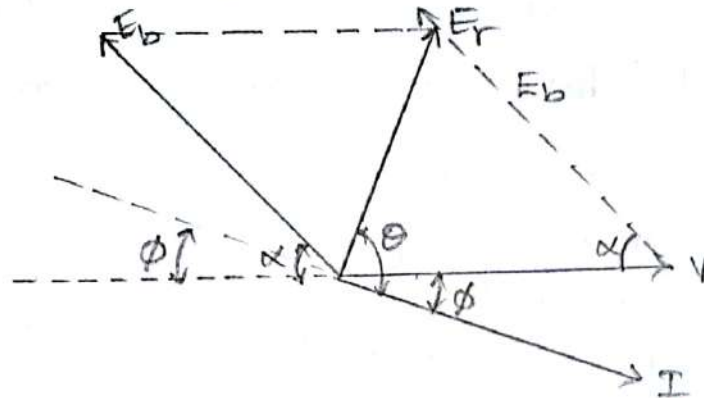


Fig. Synchronous Motor on load Vector diagram.

If the angle between stator and rotor pole centres becomes too great, due to a serious overload then the motor will pull out of synchronism and operate as an induction motor with the aid of the damper winding. The maximum value of torque which a synchronous motor can develop without dropping out of synchronism is called the "pull out torque". In most synchronous motors this is 150 to 200% of rated torque output.

Torques of a Synchronous Motor:

1. Starting torque:

- * It indicates the ability of the motor to accelerate the load. It is also sometimes called "break away torque".
- * Although the synchronous motor possesses no self-starting torque yet in modern synchronous motors, by making proper changes in the design of damper windings, almost any reasonable torque can be developed.

2. Running torque:

- * Running torque is the torque developed by motor under running conditions.
- * It is determined by the output power and speed of the driven machine.
- * Peak output power determines the maximum torque that would be required by the driven machine. The breakdown or maximum

Running torque of a motor must be greater than this value in order to avoid stalling of the machine.

3. Pull in torque:

* It pertains to the ability of the motor to pull into synchronism when changing from induction to synchronous motor operation.

4. Pull out torque:

* It is the maximum torque which the motor will develop without pulling out of step or synchronism.

* Its value varies from 1.25 to 3.5 times the full-load torque.

* Two Reactance Concept for Salient pole Synchronous Motors:

The steady state performance of salient pole synchronous motors can be predicted quite accurately by the synchronous reactance concept, however, when high degree of accuracy is required or when problems concerning transients or power stability are to be handled, the two-reactance theory is required. This theory is considered below:

* The salient pole synchronous motor has the following two axes:

- (1) Field pole axis, called the direct axis or d-axis and
- (2) The axis passing through the centre of interpolar space, called as quadrature axis or q-axis.

I_d and I_q are components of the armature current resolved along d-axis and q-axis respectively.

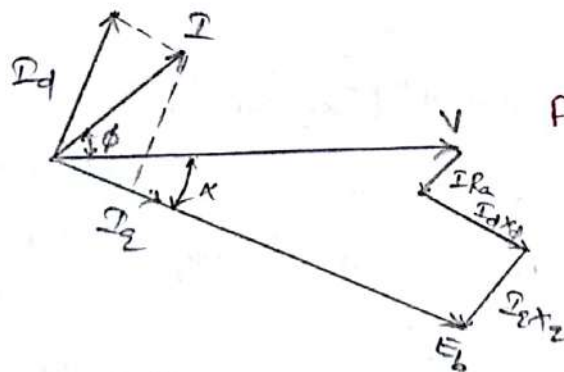


Fig 10. Phasor diagram for Synchronous motor (leading Pf) [with armature Resistance].

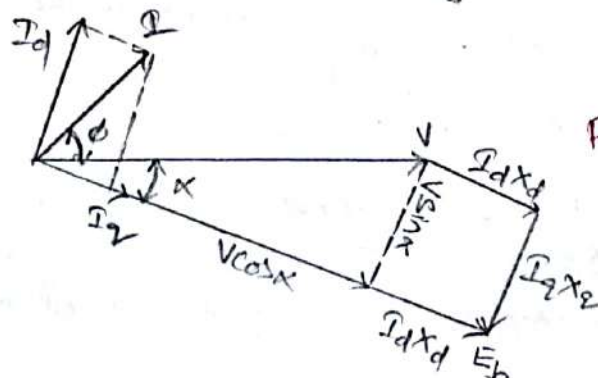


Fig 11. Phasor diagram for Synchronous Motor (leading Pf) [without arm. Resistance].

From fig(2), using E_b as reference vector

$$V \cos \alpha = E_b - I_d X_d$$

$$V \sin \alpha = I_q X_q$$

$$\therefore I_d = \frac{E_b - V \cos \alpha}{X_d} \quad \& \quad I_q = \frac{V \sin \alpha}{X_q}$$

Regardless of the axis of reference, power input is given by the product of the in-phase components of the current and voltage plus the product of the quadrature component

$$P_{in} = I_q V \cos \alpha + I_d V \sin \alpha$$

$$P_{in}(\text{per phase}) = \left(\frac{V \sin \alpha}{X_q} \right) V \cos \alpha + \left(\frac{E_b - V \cos \alpha}{X_d} \right) V \sin \alpha$$

$$P_{in} = \frac{V^2}{X_q} \sin \alpha \cos \alpha + \frac{E_b V}{X_d} \sin \alpha - \frac{V^2}{X_d} \sin \alpha \cos \alpha$$

$$= \frac{E_b V}{X_d} \sin \alpha + \frac{V^2}{2} \left[\frac{1}{X_q} - \frac{1}{X_d} \right] \sin 2\alpha$$

$$P_{in}(\text{Ph}) = \frac{E_b V}{X_d} \sin \alpha + \frac{V^2}{2} \left[\frac{1}{X_q} - \frac{1}{X_d} \right] \sin 2\alpha$$

$$\text{Total power input (3-phase power)} = 3 P_{in}(\text{Ph})$$

$$= \frac{3 E_b V}{X_d} \sin \alpha + \frac{3 V^2}{2} \left[\frac{1}{X_q} - \frac{1}{X_d} \right] \sin 2\alpha$$

$$= \frac{E_{bL} V_L}{X_d} \sin \alpha + \frac{3 V_L^2}{2} \left[\frac{1}{X_q} - \frac{1}{X_d} \right] \sin 2\alpha \quad \left[\because \begin{matrix} E_{bL} = \sqrt{3} E_b \\ V_L = \sqrt{3} V \end{matrix} \right]$$

Stability & Maximum Load angle:

$$\frac{dP_{in}}{d\alpha} = \frac{E_b V}{X_d} \cos \alpha + V^2 \left[\frac{1}{X_q} - \frac{1}{X_d} \right] \cos 2\alpha$$

The above equation gives the rate of change of power as a function of the load angle α . This called stability factor, rigidity factor or simply the stiffness of coupling.

In a smooth-cylindrical type motor motor the corresponding expression of stiffness of coupling is

$$\frac{dP_{in}}{d\alpha} = \frac{E_b V}{X_d} \cos \alpha = \frac{E_b V}{X_s} \cos \alpha \quad \left[\because \text{for cylindrical motor } X_d = X_q = X_s \right]$$

* [The stiffness factor is more for salient pole motor because of 2nd term]

Maximum load angle for salient pole motor is given by

$$\frac{E_b V}{X_d} \cos \alpha + V^2 \left[\frac{1}{X_q} - \frac{1}{X_d} \right] \cos 2\alpha = 0.$$

Solving above eqn for $\cos \alpha$

$$\cos \alpha = \frac{-E_b X_q}{4V(X_d - X_q)} + \sqrt{\frac{1}{2} + \left[\frac{E_b X_q}{4V(X_d - X_q)} \right]^2}$$

This is the cosine of the maximum load angle when armature resistance is neglected.

* Effect of Excitation on Armature current and Power factor (V-curves):

Consider a synchronous motor in which the mechanical load is constant and hence output is also constant if losses are neglected.

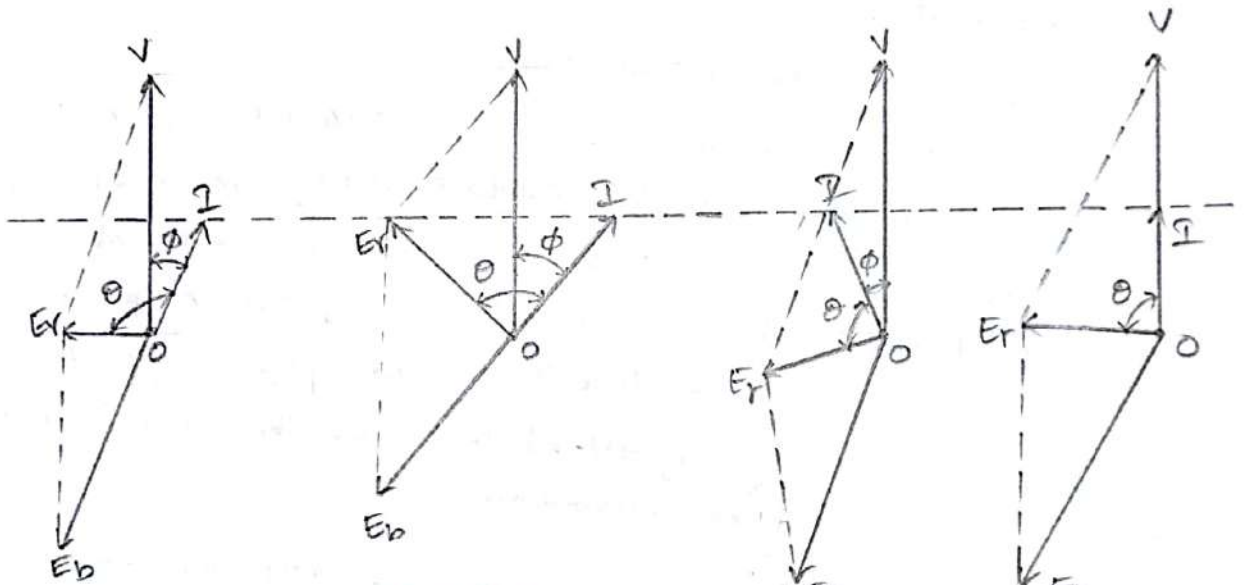


Fig (1) 100% Excitation (lag)

(2) <100% Excitation (lag)

(3) >100% Excitation (lead)

(4) UPF excitation (UPF).

Case 1: The Case for 100% excitation i.e., $E_b = V$ [fig (1)]. Here the armature current I lags behind V by a small angle ϕ . Its angle with E_r is fixed by stator constants, i.e.,

$$\tan \theta = \frac{X_s}{R_a}$$

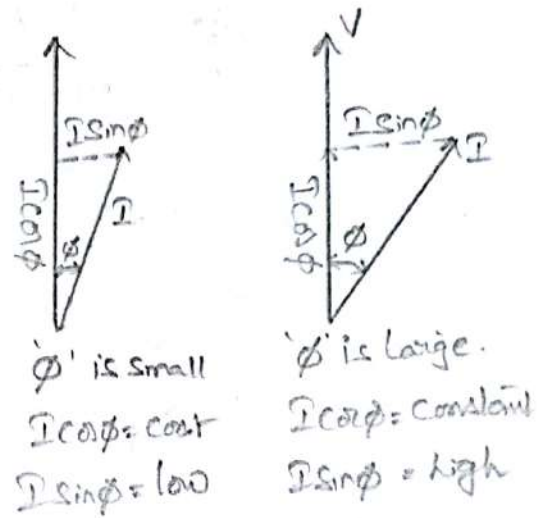
Case 2: Excitation less than 100%: fig (2)

For under excited motor, i.e., $E_b < V$. Here E_r is advanced clockwise and so is the armature current I (lags behind E_r by a fixed angle θ). We find that:

The magnitude of I is increased, but its power factor is decreased ($\because \phi$ has increased).

— Since input as well as V are constant, hence the power component of I , i.e., $I \cos \phi$ remains the same as before, but wattless component $I \sin \phi$ is increased.

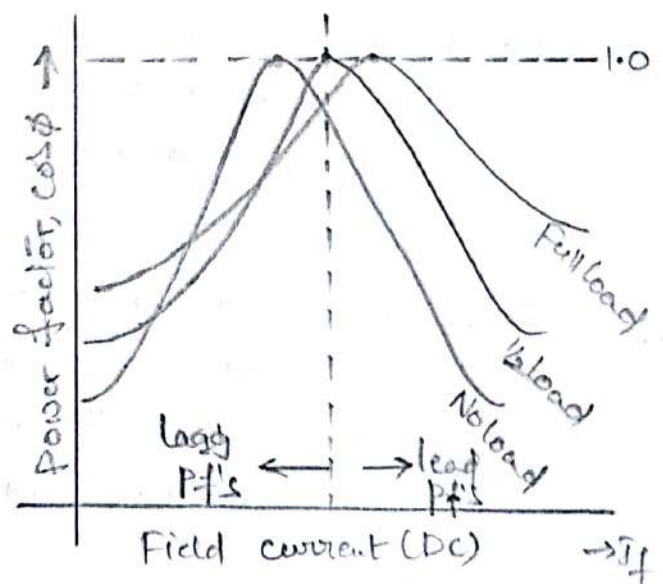
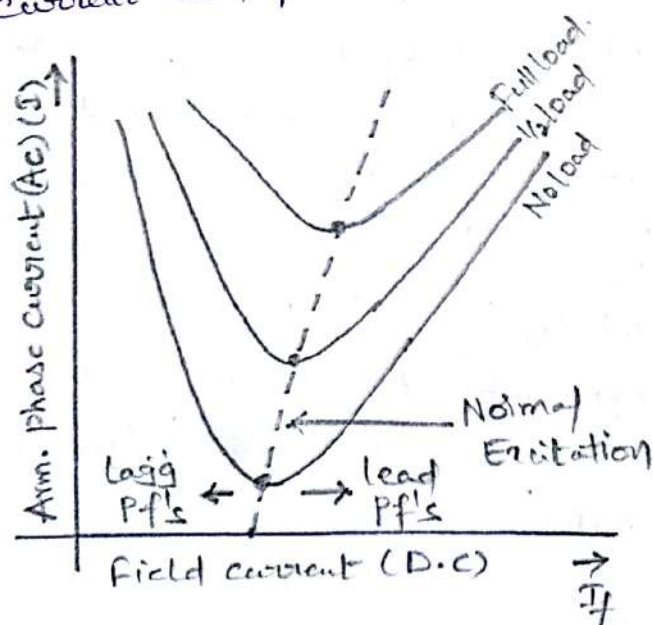
Hence, as excitation is decreased, I will increase, but power factor will decrease so that power component, i.e., $I \cos \phi$ will remain constant. The locus of the extremity of current vector would be horizontal straight line.

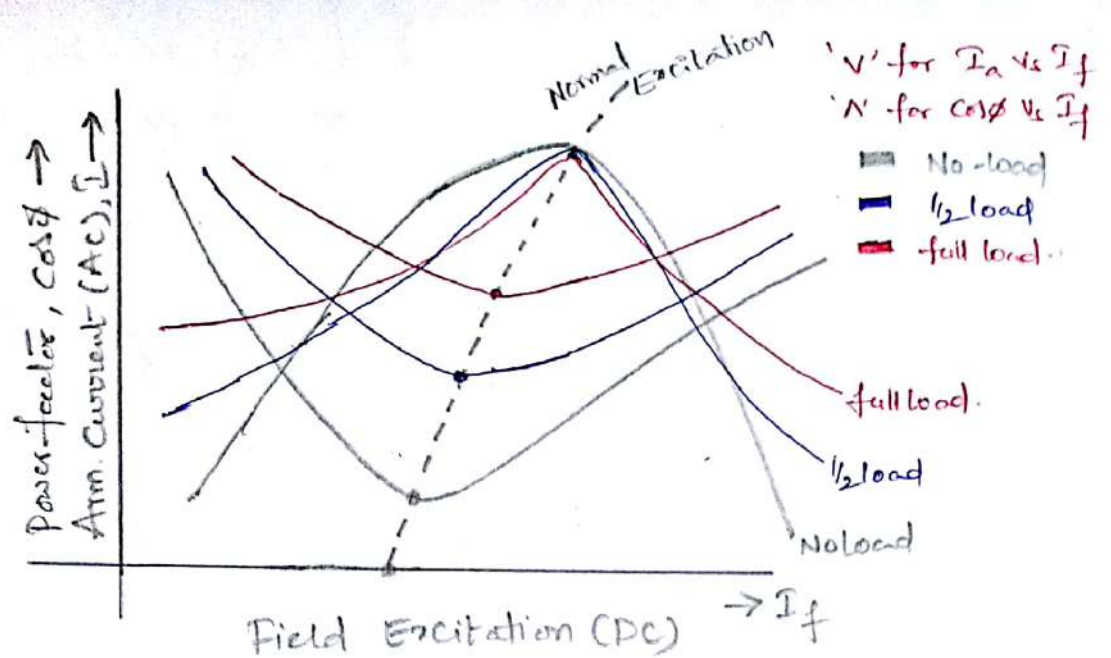


Case 3: Excitation greater than 100%:

In fig(3) excitation is greater than 100%, i.e., $E_b > V$ (i.e., motor is over excited). Here the resultant voltage vector E_r is pulled over excited and so is I . It may be noted that now motor is drawing a leading current. It may also happen for some value of excitation, that I may be in phase with V i.e., power factor is unity (fig. (4)). At that time the current drawn by motor would be minimum.

From the above analysis, the effect of excitation on armature current and power factor as follows:





Laboratory Experimental Connection Diagram for 'V' curves:

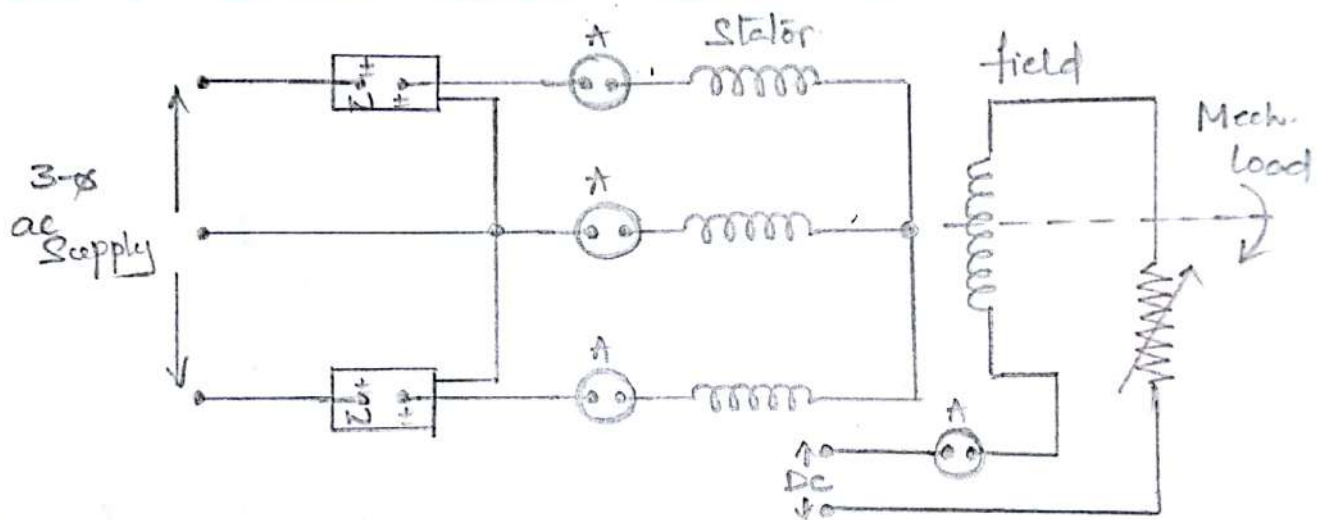


Fig. Experimental Connection diagram for 'V' curves of Sync. Motor.

Prob: A 400V, 1- ϕ Synchronous motor gives a net output mechanical power of 7.35kW and operates at 0.92 pf lagging. Its effective resistance is 0.7 Ω . If the iron & mechanical losses are 550W and excitation losses are 750W, Calculate:

(i) Armature Current and (ii) Commercial efficiency.

Sol: Given data Voltage, $V = 400V$
 Net O/p Mech. power = 7.35kW
 power factor = 0.92 lag
 Arm. Resistance = 0.7 Ω
 Stray losses = 550W
 Excitation losses = 750W

$$\text{Power input} = VI \cos \phi = 400 \times I \times 0.92 = 368I$$

$$\begin{aligned} \text{Power o/p} &= \text{Power i/p} - \text{losses} \\ &= \text{Power i/p} - [\text{Cu loss} + \text{shay losses}] \end{aligned}$$

$$7350 = 368I - [I^2 \times 0.7 + 550]$$

$$7350 = 368I - 0.7I^2 - 550$$

$$0.7I^2 - 368I + 7900 = 0$$

$$I = 503.29 \text{ A or } 22.42 \text{ A}$$

$$I = 22.42 \text{ A [Because of 1-}\phi\text{]}$$

$$\boxed{\text{Armature current, } I = 22.43 \text{ A}}$$

Commercial efficiency:

$$\begin{aligned} \text{i/p power including Excitation losses} &= 400 \times 22.43 \times 0.92 + 750 \\ &= 9004.24 \text{ W} \end{aligned}$$

$$\eta_c = \frac{\text{o/p}}{\text{i/p}} \times 100 = \frac{7350}{9004.24} \times 100$$

$$\boxed{\% \eta_c = 81.6\%}$$

Prob: A 2200V, 3- ϕ Star-connected Synchronous motor has a resistance of 0.22 Ω /ph and Synchronous reactance of 2.4 Ω /ph. The motor is operating at 0.6 powerfactor leading with a line current of 180A. Determine the value of the generated emf/ph.

Sol: Given that Voltage, $V = 2200 \text{ V (line)}$
 $= 1270 \text{ V (ph)}$

$$R_a = 0.22 \Omega/\text{ph}, X_s = 2.4 \Omega/\text{ph}$$

$$\text{Motor powerfactor, } \cos \phi = 0.6$$

$$\phi = \cos^{-1}(0.6) = 53.13^\circ$$

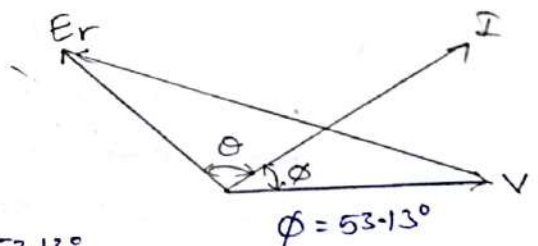
$$\text{Internal Pf angle, } \theta = \tan^{-1}\left(\frac{X_s}{R_a}\right) = \tan^{-1}\left(\frac{2.4}{0.22}\right) \Rightarrow \theta = 84.76^\circ$$

$$\text{Line current, } I = 180 \text{ A}$$

From the phasor diagram

$$\boxed{E_b = \sqrt{V^2 + E_f^2 - 2VE_f \cos(\theta \pm \phi)} \quad \begin{array}{l} + \text{ for lead} \\ - \text{ for lag} \end{array}}$$

$$E_f = IZ_s = 180 \times (0.22 + j2.4) = 433.8 \angle 84.76^\circ \text{ V}$$



For lead power factor

$$E_b = \sqrt{V^2 + E_r^2 - 2VE_r \cos(\theta + \phi)}$$

$$= \sqrt{1270^2 + 433.8^2 - 2 \times 1270 \times 433.8 \times \cos(84.76 + 53.31^\circ)}$$

$$E_b = 1618.89 \text{ V (ph)}$$

$$E_b = 2803.9 \text{ V (line)}$$

Prob: A 6600V, 3- ϕ , star connected Synchronous motor draws a full-load current of 80A at 0.8 pf leading. The armature resistance is 2.2 Ω and reactance 22 Ω per phase. If the stray losses of the machine are 3200W, find:

(i) EMF induced (ii) output power and (iii) Efficiency of the M/c.

$$[4962.5 \text{ V}, 686.178 \text{ kW} \times 93.79\%]$$

Prob: A 11kV, 3- ϕ star connected Synchronous motor draws a current of 45A. The effective resistance & synchronous reactance per phase are 0.9 Ω & 28 Ω respectively. Calculate the power supplied to the motor & induced emf for a power factor of:

(i) 0.8 lagging & (ii) 0.8 leading.

$$[685.892 \text{ kW}, 5647.5 \text{ V}, 9781.7 \text{ V}, 685.892 \text{ kW}, 7148.6 \text{ V}, 12381.7 \text{ V}]$$

* Synchronous Condensers:

* Several Synchronous motors are manufactured without any shaft extensions whatever, ostensibly designed solely for power-factor correction and to be operated without any mechanical load.

"Any over-excited Synchronous motor operated without any mechanical load", however, may be classed as a Synchronous - Condenser or Synchronous Capacitors.

* As Compared with a Synchronous motor with equal armature voltage and current ratings, a Synchronous Condenser requires more copper in the field winding to carry the increased field current. The Synchronous Condensers does not require so large shaft and bearings as the Synchronous motor because no shaft torque is required.

* Synchronous Condensers are sometimes operated at power factors ranging

from lagging through unity to leading for voltage control. When applied in this manner a synchronous condenser is called a synchronous reactor.

* It is customary not to attempt correction of the powerfactor of the system all the way to a unity powerfactor. There is an economic reason for this, despite the fact that large, high capacity synchronous capacitors are available and for the same KVA rating are usually less expensive than synchronous motors. The capacitors are less expensive because:

- (i) They are started and run without load, requiring no special heavy cage windings, and
- (ii) They require smaller shaft diameters and lighter bearings, although their field windings are somewhat heavier.

Prob: A 440V, 50Hz, 3- ϕ circuit takes 18A at a lagging powerfactor of 0.8. A synchronous motor is used to raise the powerfactor to unity. Calculate the KVA input to the motor, and its powerfactor when driving a mechanical load of 6KW. The motor has an efficiency of 88%.

Sol: Given that, Voltage, $V = 440V$ (line)
 $= 254V$ (Ph)

Power factor of ckt = 0.8 lagging
 (Old Pf)

New power factor of ckt = 1
 (New Pf)

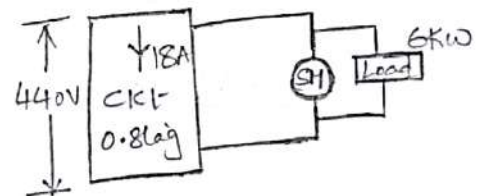
The mech. load on Sync. Motor = 6KW.
 Sync. Motor, $\eta = 88\%$.

$$\begin{aligned} \text{KVAR drawn by 3-}\phi \text{ circuit} &= KV \times \sin \phi \\ &= \sqrt{3} V_L I_L \sin \phi = \sqrt{3} \times 440 \times 18 \times 0.6 \\ &= 8.23 \text{ KVAR} \end{aligned}$$

$$\text{Power Supplied by Sync. Motor, } P = \frac{\text{O/P in kW}}{\eta} = \frac{6 \text{ K}}{0.88} = 6.82 \text{ kW.}$$

Required reactive power to increase the powerfactor unity is Q_c ,

$$Q_c = 8.23 \text{ KVAR (KVAR is zero, Pf is upf).}$$



Synchronous Motor rating, $S = P - jQ$

$$S = (6.82K - j8.23K) VA$$

$$S = \sqrt{6.82K^2 + 8.23K^2}$$

$$S = 10688.55 VA$$

Synchronous Motor rating, $S = 10.68855 KVAR.$

Prob: A 3- ϕ Synchronous motor absorbing 60kW is Connected in parallel with a factory load of 240kW having a lagging power factor of 0.8. If the Combined load has a P.f of 0.9, what is the value of leading KVAR Supplied by the motor and at what power factor is it working?

Sol: Given data 3- ϕ sync. Motor - $P_s = 60KW$

factory load, $P_f = 240KW$ at $\cos\phi_1$ 0.8 lag P.f

Combined power factor, $\cos\phi_2 = 0.9$

factory load: $P_f = 240KW$

$$Q_f = P_f \tan\phi_1 = 240K \times \tan[\cos^{-1} 0.8] = 180 KVAR$$

$$\text{factory load, } S_f = P_f + jQ_f = (240 + j180) KVA$$

Sync. Motor: $P_s = 60KW$

$$Q_s = ?$$

Combined load: $P = P_f + P_s = 240K + 60K = 300KW$

$$Q = P \tan\phi_2 = 300K \times \tan[\cos^{-1} 0.9] = 145.3 KVAR$$

Total load, $S = P + jQ$

$$S = (300 + j145.3) KVA$$

From the above calculation, Compensated reactive power

$$Q_s = Q_c = Q_f - Q = 180 - 145.3 = 34.7 KVAR$$

$$\therefore \text{Sync. Motor, } S_s = P_s - jQ_s = (60K - j34.7K) VA = S_s$$

$$\text{Sync. Motor P.f} = \frac{P_s}{S_s} = \frac{60K}{\sqrt{60K^2 + 34.7K^2}} = 0.866 \text{ lead} = \text{P.f}$$

* Hunting of Synchronous Motors:

When the mechanical load is constant, the motor settles down to an absolutely constant speed with the torque angle fixed by the particular delivered horsepower. Should the load be changed, however, the motor speed changes momentarily until the torque angle adjusts itself to the new horsepower, if the load increases, the slips backwards to an increased torque angle, while a load reduction causes the motor to advance to a smaller torque angle. But because of the moment of inertia of the rotating parts, the motor overshoots the final position, slowing down or speeding up more of than it should. In slowing down as a result of an increased load, for example, it passes the proper torque angle giving up some of the kinetic energy; under this condition, the motor develops more torque than it requires and speeds up. Acceleration to locate the correct torque angle causes the motor to move forward a little more than it should; this results in less than the required torque, so that the motor slows down again. It may be understood that this periodic speed change is only momentary while the motor is attempting to settle down to a correct torque angle and that it goes on while the average speed is constant.

This rapid forward & backward motion of the motor as it revolves at the average constant speed is called "hunting" (i.e., the motor may be said to hunting for its correct position with respect to the absolutely constant speed of the revolving field.)

* Hunting is an objectionable characteristic of all synchronous motors, since it produces severe mechanical stresses as well as great variations in current and power taken by the motor.

* Fortunately, the very same pole-face of squirrel cage that provides the machine with its starting torque is instrumental in damping the oscillations. Its effectiveness in doing this, however, depends upon the resistance of the squirrel cage; the lower resistance the stronger is the damping action. But since a high squirrel cage

resistance is necessary; if the Synchronous motor is to have a good starting torque, it is customary to employ a compromise value for a cage. The term damping is generally applied to the squirrel cage in connection with its damping action.

This damping action results because by Lenz's law any change in flux linking the damper as it attempts to oscillate back and forth causes a current to flow in the cage; this current then flows in such a direction as to oppose a change in the flux that normally links the armature and the rotor.

* Comparison between Synchronous and Induction Motors:

S.No.	Synchronous Motor	Induction Motor
1.	It is inherently not self-starting and some external means are required for its starting.	It has got self starting torque and no special means are required for starting.
2.	Requires D.C. Excitation	Does not require Dc excitation.
3.	Speed control not possible	Speed can be controlled but to small extent.
4.	Its average speed is constant and independent of load.	Its speed falls with the increase in load and is always less than synchronous speed.
5.	It can be operate under a wide range of power factors, both lag and leading.	It operates at only lag power factor, which becomes very poor at light loads.
6.	Its torque is less sensitive to change in supply voltage	Its torque is more sensitive to change in supply voltage.
7.	Breakdown torque is proportional to the supply voltage.	Breakdown torque is proportional to the square of supply voltage.
8.	More complicated & more costly comparatively.	More simple and less costly comparatively.
9.	Employed for supplying mech. load as well as for power factor improvement.	Employed for supplying mech. load only.